1. Vertical beam size & 2. Nonlinear Dynamics

- Emittance and equilibrium beam size
- Resonances
- Coupling correction
- Nonlinear dynamics

Horizontal beam size & emittance

- Individual particles execute betatron oscillations about the beam center:

$$x_\beta(s) = \sqrt{\varepsilon_A}\sqrt{\beta_x(s)}\,\cos(\varphi_x(s) + \varphi_0)$$

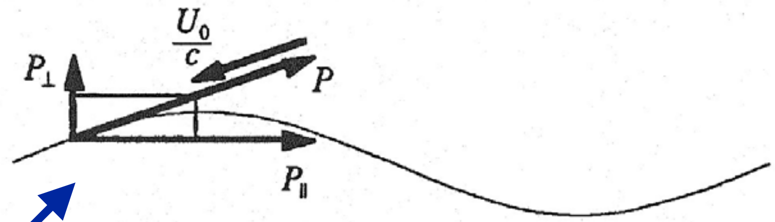
- where $\beta_x(s)$ is an amplitude that varies with position around the ring, and ε_A is a constant.
- The electrons in an electron bunch are distributed equally in betatron phase and have a Gaussian transverse distribution.
- The horizontal emittance specifies the width of this Gaussian:

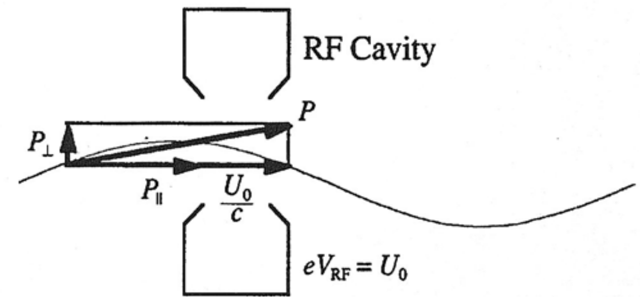
$$\varepsilon_x = \sigma_x^2/\beta_x(s)$$

- The emittance is determined by the balance between damping and excitation of oscillations from synchrotron radiation.

Radiation damping & Quantum excitation

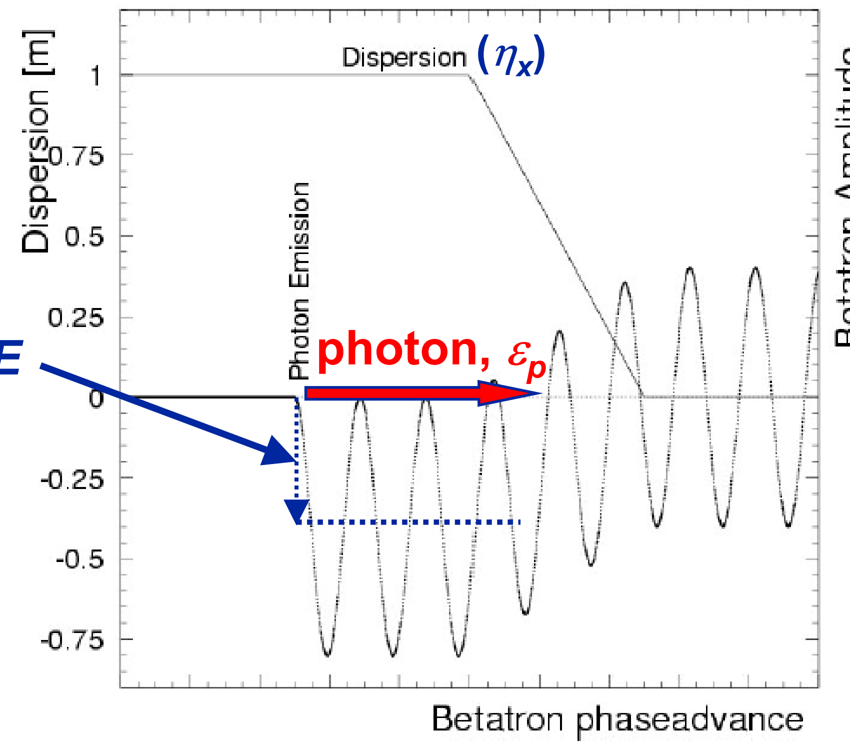
- **Radiation damping:**
  - Longitudinal & transverse momentum is lost to synchrotron radiation
  - Only longitudinal momentum replaced by RF cavity





- **Quantum excitation:**
  - A photon radiated at some place with dispersion shifts the closed orbit for the electron.
  - Electron oscillates about new closed orbit: excitation of betatron oscillation.

$\Delta x_{c.o.} = -\eta_x \varepsilon_p / E$



Emittance and beam size

- Balance between damping and quantum excitation leads to an equilibrium emittance:

$$\varepsilon_x \propto \int_{dipoles} ds(\gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta'^2_x) \qquad (1)$$

- The total horizontal beam size has a contribution from β oscillations as well as a contribution from energy spread:

$$\sigma_x = \sqrt{\beta_x \varepsilon_x + \eta_x^2 \left(\frac{\sigma_E}{E} \right)^2} \qquad (2)$$

- In ideal accelerator η_y = 0, so σ_y is negligibly small.
- In reality, vertical beam size has 3 contributions:
  - Quantum excitation of y_β-oscillations as in eqn. 1 above (all $x \rightarrow y$) through small η_y.
  - Orbit spread due to $\sigma_E * \eta_y$ as in eqn. 2 above.
  - Coupling of x_β-oscillations into y_β.

Review

- So far we've looked at three solutions to eqn. of motion:

1.

$$x''+K_x(s)x=0$$ → β oscillations

$$x_\beta(s)=\sqrt{\varepsilon_A\beta(s)}\cos(\varphi(s)-\varphi_0)$$

2.

$$y''+K_y(s)y=\theta\delta(s-s_0)$$ → Closed orbit shift

$$y_{c.o.}(s)=\theta\frac{\sqrt{\beta(s)\beta(s_0)}}{2\sin(\pi\nu)}\cos(|\varphi(s)-\varphi(s_0)|-\pi\nu)$$

3.

$$x''+K_x(s)x=\frac{\delta_\varepsilon}{\rho}$$ → Dispersion

$$x(s)=\eta\delta_\varepsilon$$

- Now a fourth:

$$x''+K_x(s)x=-K_s(s)y \qquad y''+K_y(s)y=-K_s(s)x$$ → Coupling

(where $K_x=-K_y(s)=-\frac{1}{B\rho}\frac{\partial B_y}{\partial x} \qquad K_s(s)=\frac{1}{B\rho}\frac{\partial B_x}{\partial x}$)

Difference coupling resonance (approximate math)

- The equation $y'' + K_y(s)y = -K_s(s)x$ is like a driven harmonic oscillator.
- If a harmonic oscillator is driven on resonance, get large oscillations – large vertical beam size, in this case.
- On resonance if vertical drive is at the vertical tune:

$$K_s(s)x \sim \exp\left(i2\pi\nu_y \frac{s}{L_0}\right) \quad \text{i.e.} \quad K_s(s) \sim \exp\left(i2\pi(\nu_y - \nu_x)\frac{s}{L_0}\right)$$

- But $K_s(s)$ is periodic in L_0 (the ring circumference), so it only has Fourier terms period in L_0:

$$K_s(s) \sim \exp\left(i2\pi n \frac{s}{L_0}\right), \quad n = \text{integer}$$

- So y_β driven on resonance if:
  1. $\nu_x - \nu_y = n$ (integer) this is a difference coupling resonance
  2. $K_s(s)$ has a spatial Fourier component at n to drive this resonance

More resonances

- Skew quadrupoles, $K_s(s)$, can also drive the sum coupling resonance, if:
  1. $\nu_x + \nu_y = n \ \text{(integer)}$
  2. $K_s(s)$ has a spatial Fourier component at n to drive this resonance
- Skew quadrupoles can also couple η_x to create η_y.

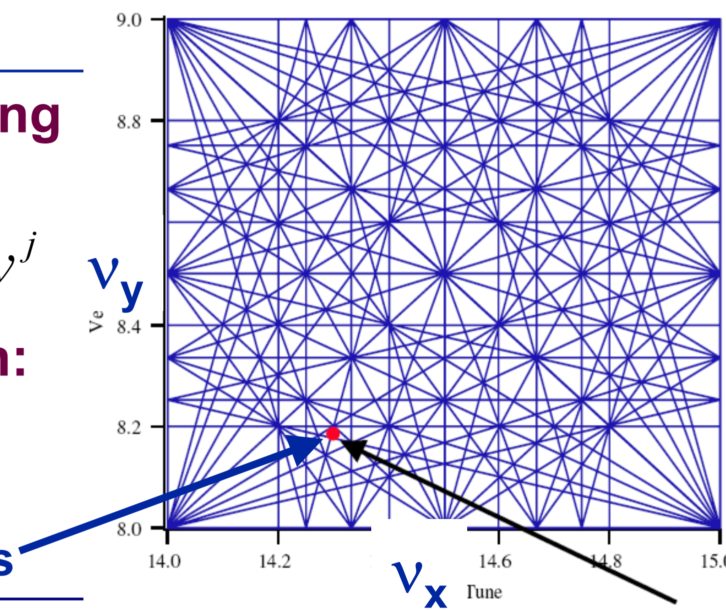
$$y'' + K_y(s)y = -K_s(s)(x_\beta + \eta_x \delta_\varepsilon), \quad K_s \eta_x \delta_\varepsilon \text{ drives } \eta_y$$

- This analysis extends to nonlinear driving terms

$$x'' + K_x(s)x = A_{i,j} x^i y^j, \quad y'' + K_y(s)y = B_{i,j} x^i y^j$$

- Generating nonlinear resonances, when:

$$n\nu_x + m\nu_y = N \qquad (n, m, N \text{ integers})$$



Skew quadrupole corrector distribution

When choosing where to locate skew quadrupoles, must distribute them to correct both phases of coupling resonances and vertical dispersion:

- **Distribute in difference coupling resonance phase**

$$\kappa = \frac{1}{4}\pi \int ds K_s \sqrt{\beta_x \beta_y} e^{i\Phi_D} \qquad \frac{\Phi_D(s)}{2\pi} = (\mu_x(s) - \mu_y(s)) - \frac{s}{C}(\nu_x - \nu_y - N)$$

- **In sum coupling resonance phase**

$$\frac{1}{4}\pi \int ds K_s \sqrt{\beta_x \beta_y} e^{i\Phi_S} \qquad \frac{\Phi_S(s)}{2\pi} = (\mu_x(s) + \mu_y(s)) - \frac{s}{C}(\nu_x + \nu_y - M)$$

- **And in η_y phase**

$$\eta_x \sqrt{\beta_y} e^{i\Phi_{\eta_y}} \qquad \frac{\Phi_{\eta_y}(s)}{2\pi} = \mu_y(s) - \frac{s}{C}(\nu_y - N)$$

  - **Need some skew quadrupoles at non-zero η_x**

Resonance Description of Global Coupling

- Global coupling is typically described using a resonance theory
- Difference coupling resonance

$$\kappa = \frac{1}{4\pi} \int ds\, K_s \sqrt{\beta_x \beta_y}\, \mathrm{e}^{i\phi_D}$$

$$\frac{\phi_D}{2\pi} = \mu_x(s) - \mu_y(s) - \frac{s}{C}\Delta_\mathrm{r} \qquad \Delta_\mathrm{r} = (\nu_x - \nu_y - N)$$

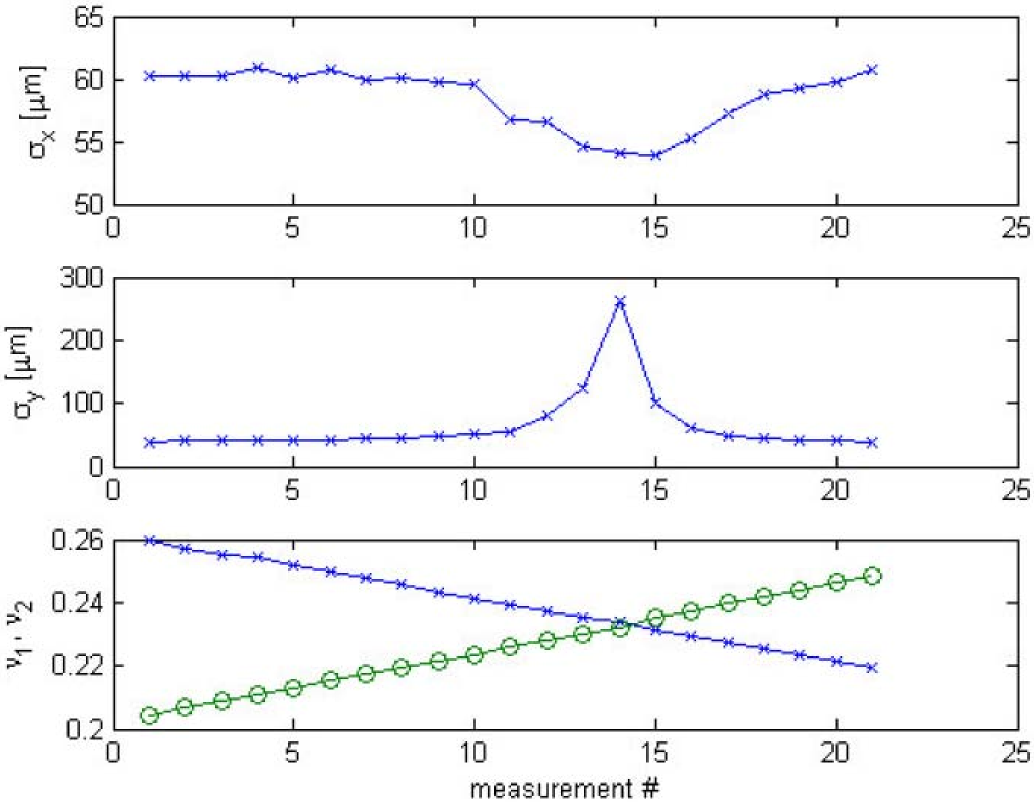
- Vertical emittance near difference resonance:

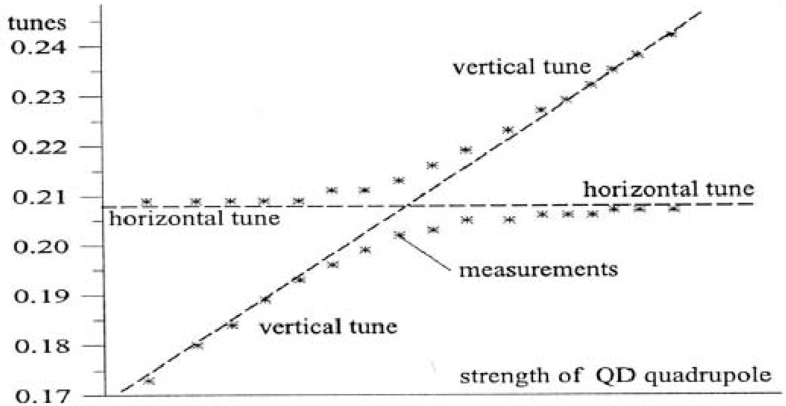
$$\frac{\varepsilon_y}{\varepsilon_x} = \frac{|\kappa|^2}{|\kappa|^2 + \Delta_\mathrm{r}^2 / 2}$$

κ is resonance strength, Δ_r is distance from resonance.

Scan of difference resonance

- There are sum resonances as well (phase advance proportional to sum of horizontal and vertical phase advance) and of course higher order resonances.
- One can create orthogonal knobs of skew quadrupoles directly acting on one of those coupling resonances



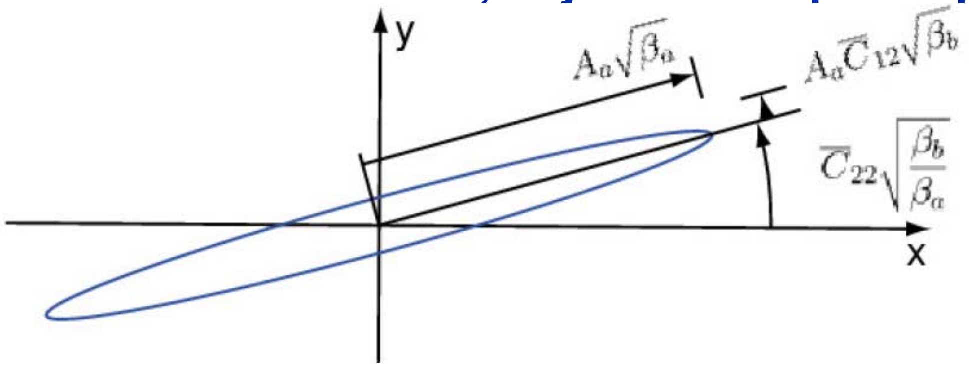


- Minimum tune split (on resonance):

$$(\nu_x - \nu_y)_{\min} = 2|\kappa|$$

More advanced coupling correction algorithms

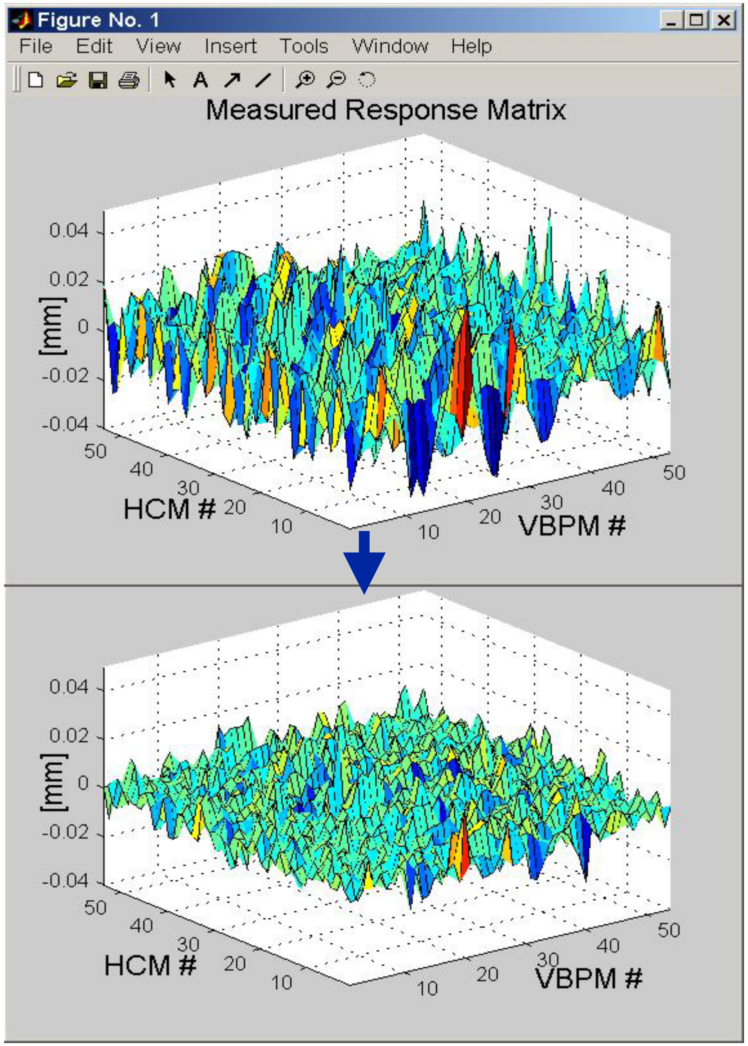
- **Using digitized turn-by-turn driven oscillations (CESR)**
  - with no coupling, if you drive the horizontal tune, you see only horizontal oscillations
  - with coupling, see some vertical oscillation as well.
  - measure vertical oscillations at all BPMs; adjust skew quadrupoles to minimize them.



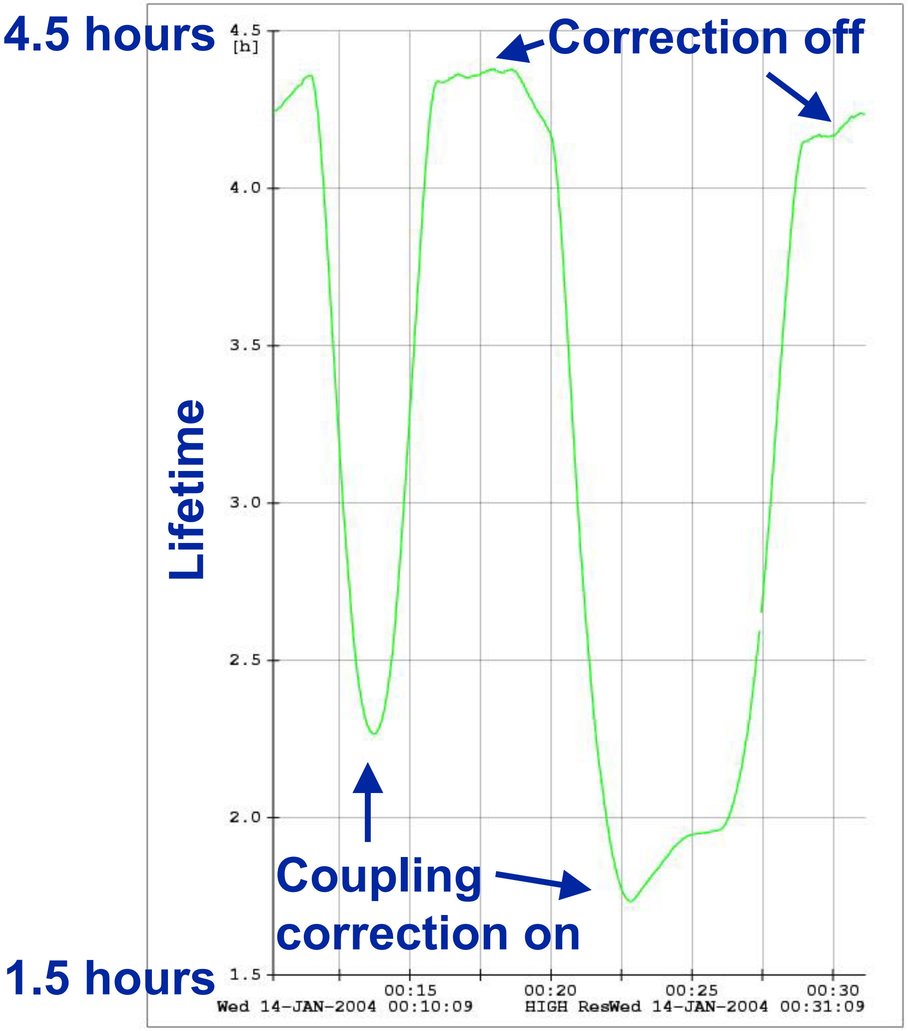
- **Using closed orbit response – LOCO (NSLS, ESRF, ALS, SPEAR3)**
  - Correcting closed orbit coupling not the same as correcting betatron coupling, but simulations and experiments show it's close enough.
  - With LOCO, can simultaneously correct η_y to get minimum σ_y.

Coupling & η_y correction, LOCO

Minimize η_y and off-diagonal response matrix:



Lifetime, 19 mA, single bunch



Simulation of coupling correction with LOCO

- Use accelerator toolbox (Andrei Terebilo), Matlab and LOCO (James Safranek, Greg Portman) for simulations
- Use random skew error seeds
- Try to find effective skew corrector distributions and to optimize correction technique in simulation
- Used two correction approaches:
  1. Response Matrix fitting – 'deterministic', small number of iterations
  2. Direct minimization (nelder-simplex, …) – easy to do on the model, difficult on real machine

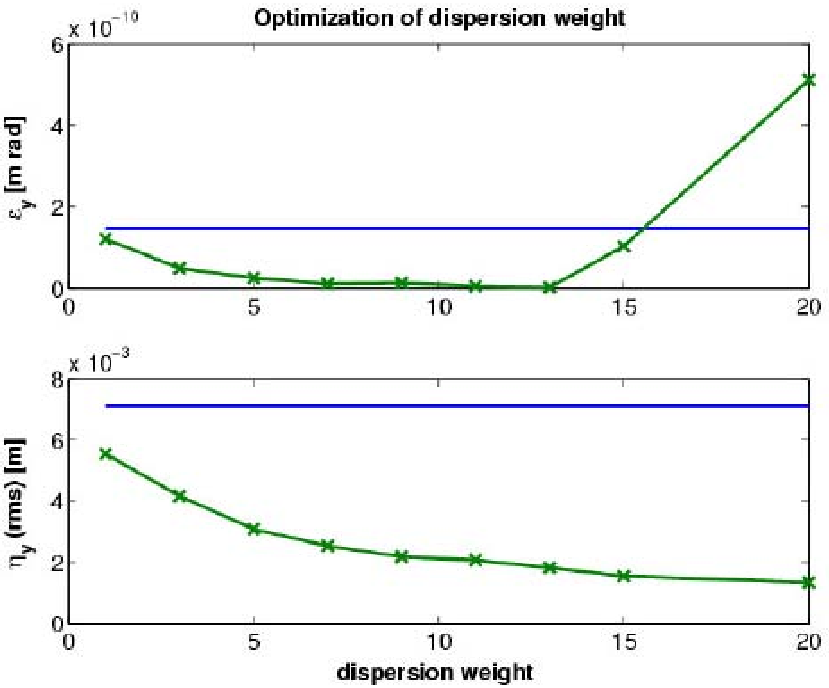
– Surprisingly both approaches gave about the same performance in the model calculations

– For response matrix analysis you have to optimize several parameters of the code as well (weight of dispersion, number of SVs, use of effective model/full model …)

Thanks Christoph

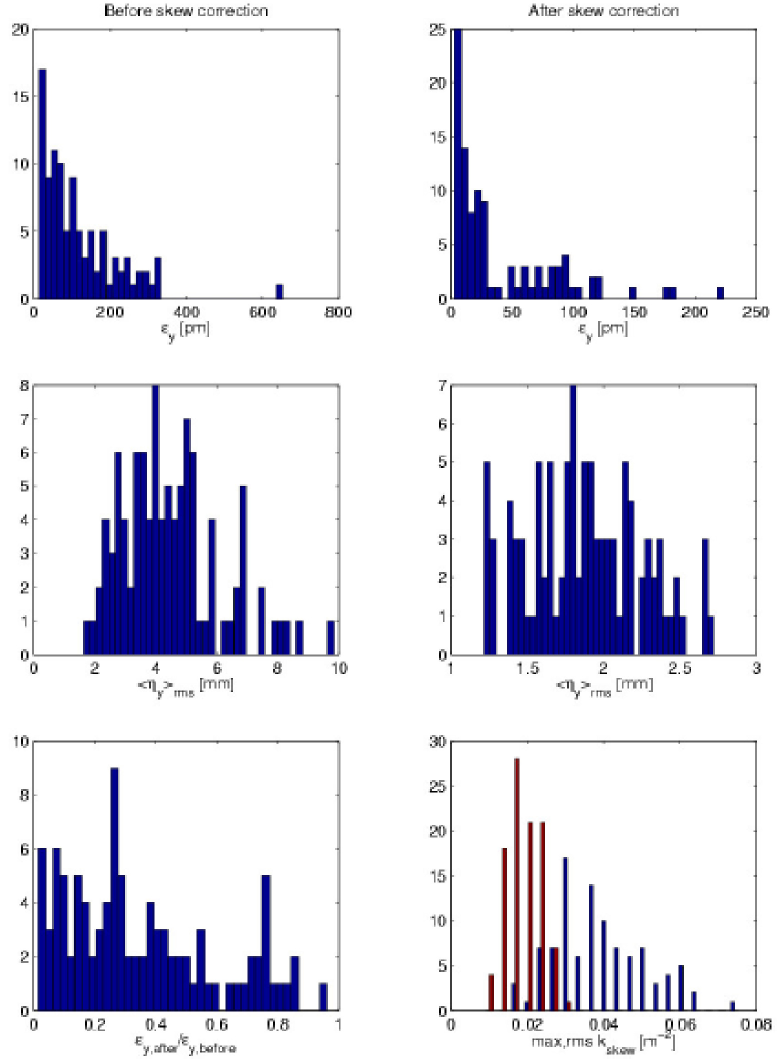
Weight of dispersion in LOCO fit

- The relative contribution of vertical dispersion and coupling to the vertical emittance depends on the particular lattice (and the particular error distribution).
- Therefore the optimum weight for the dispersion in the LOCO fit has to be determined (experimentally or in smulations).
- The larger the weight factor, the better the vertical dispersion gets corrected, but eventually the coupling 'explodes'.
- Set weight to optimum somewhat below that point.
- Outlier rejection tolerance might be important parameter as well.



Finding an Effective Skew Quadrupole Set

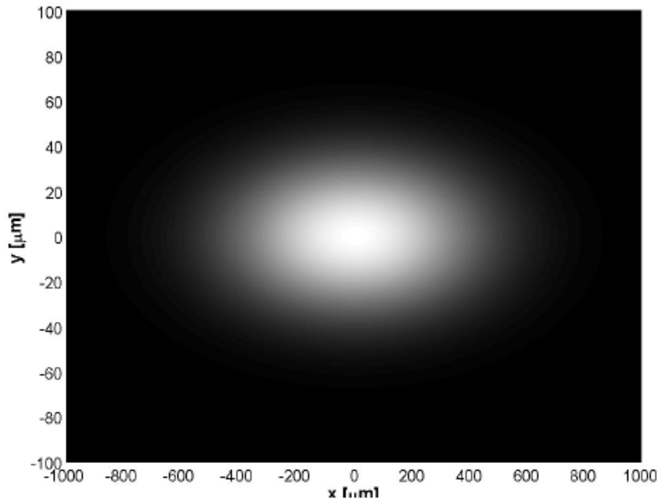
- To find an effective skew quadrupole distribution, we used several correction methods, first in simulations – best method was orbit response matrix fitting (using LOCO)
- Predictive method, can be easily used on real machine
- Issues are:
  - Cover set of phases relative to dominant coupling resonance(s)
  - Magnets should be distributed around the ring in order to avoid excessive local coupling/vertical dispersion
  - Need different values of dispersion/beta function to be effective both for coupling and vertical dispersion correction
- Set of 12 skew quadrupoles was reasonably efficient

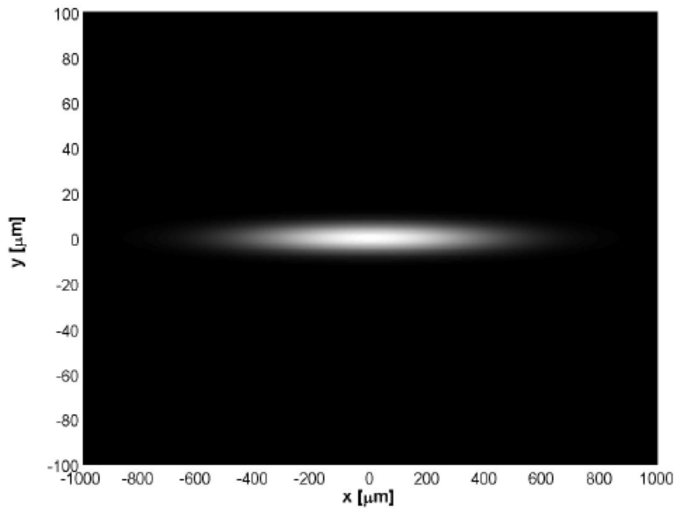


LOCO coupling correction ALS/NSLS

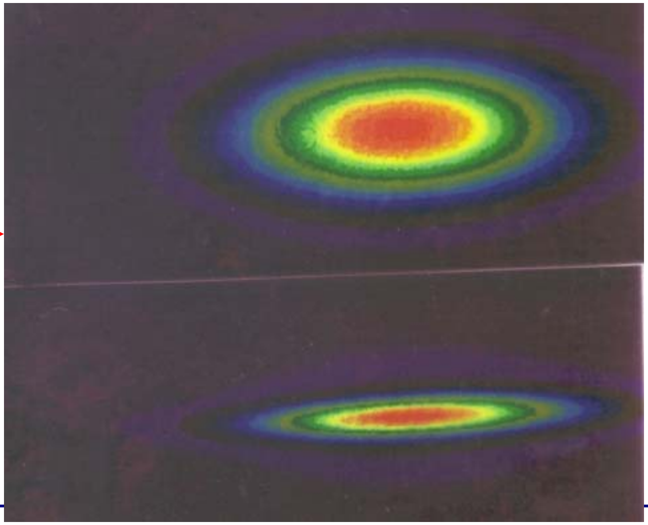
ALS

- Achieved an emittance reduction from 150 pm (routine ALS operation) to about 5 pm (pictures on the right illustrate size reduction for insertion device straights)
- This was a world record in 2003 and about the NLC damping ring design value
- Correspondingly the brightness increases by factor 30 (for hard x-rays – because of diffraction limit less for soft x-rays)





NSLS X-Ray Ring



Further reading on coupling correction

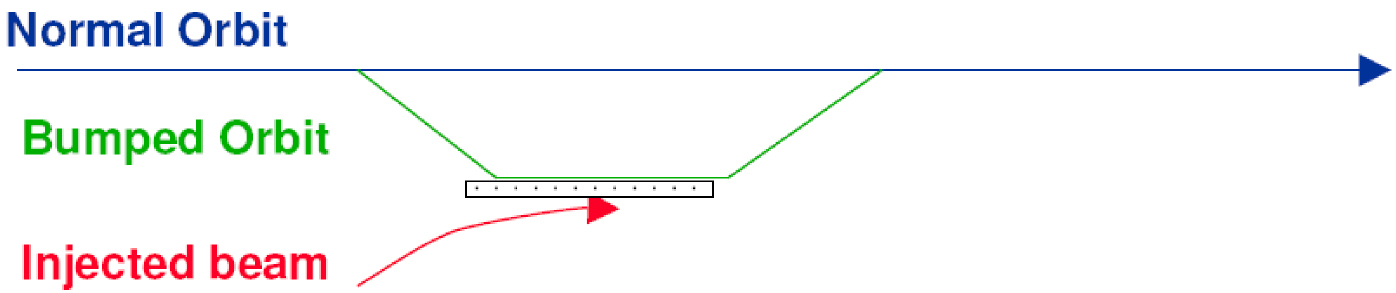
- Guignard, CERN 76-06 1976
- (De Ninno & Fanelli, PRST-AB, Vol 3, 2000).
- K. Ohmi et al., PRE 49, No 1, 1994
- D. Sagan and D. Rubin, PRST-AB, Vol 2, 1999
- D. Sagan et al. PRST-AB, Vol. 3, 2000.
- J. Safranek, and S. Krinsky, PAC'93 and AIP Proc. 315, 1993.
- J. Safranek, NIM A 388, p 27, 1997.
- C. Steier, and D. Robin, EPAC'00.
- P. Nghiem, and Tordeux, Coupling correction for the ESRF, SOLEIL internal report, 1999.
- R. Nagaoka, EPAC'00.
- R. Nagaoka, and L. Farvacque, PAC'01.
- K. Kubo, et al., Phys. Rev. Lett. 88:194891 (2002)
- C. Steier, et al., 'Coupling Correction and …', PAC 2003

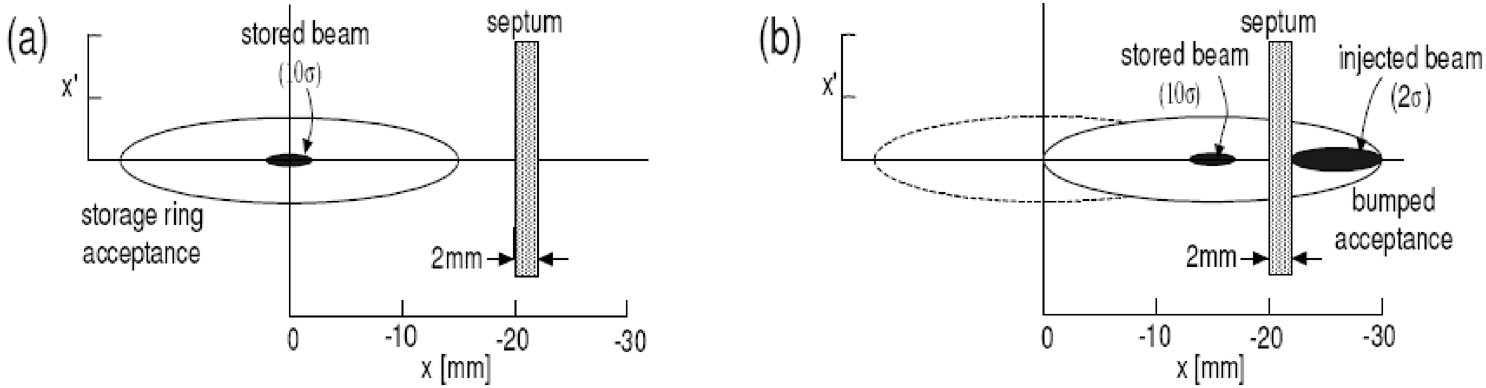
Nonlinear dynamics - motivation

- Motion of particles at large amplitudes impacts the performance of the storage ring

- Particle loss:
  - Injection efficiency
    - Longer injection times
    - Increased radiation levels
  - Lifetime
    - More frequent fills
    - Faster current loss (varying brightness/photon optics thermal load)

Injection efficiency





- **Storage ring dynamic aperture must be large enough to capture sufficient fraction of injected beam.**

Storage ring lifetime

- Elastic Scattering

$$\frac{1}{\tau_{el}} \propto \frac{1}{E^2} \times \left(\frac{\beta_x}{\Delta_x^2} \langle P\beta_x \rangle + \frac{\beta_y}{\Delta_y^2} \langle P\beta_y \rangle \right) \qquad (1)$$

- Touschek Effect

$$\frac{1}{\tau_{tou}} \propto \frac{1}{E^3} \frac{I_{bunch}}{V_{bunch}\sigma'_x} \frac{1}{\varepsilon} f\left(\varepsilon, \sigma'_x, E\right) \qquad (2)$$

- Quantum Lifetime

$$\frac{1}{\tau_q} \propto \frac{\Delta^2}{\sigma^2} \times \exp\left(-\frac{\Delta^2}{2\sigma^2} \right) \qquad (3)$$

- Inelastic Scattering

$$\frac{1}{\tau_{inel}} \propto \langle P \rangle \times \ln(\varepsilon) \qquad (4)$$

$$\frac{1}{\tau} = \frac{1}{\tau_{el}} + \frac{1}{\tau_{tou}} + \frac{1}{\tau_{ql}} + \frac{1}{\tau_{inell}}$$

Resonances

- Nonlinear driving terms in the equation of motion

$$x''+K_x(s)x = A_{i,j}x^i y^j, \qquad y''+K_y(s)y = B_{i,j}x^i y^j$$

- Generate nonlinear resonances, when:
  1. $n\nu_x + m\nu_y = N \qquad (n, m, N \text{ integers})$
  2. A_{ij} has the N^{th} spatial Fourier component to drive this resonance
- Benefit of periodicity, resonance reduction:

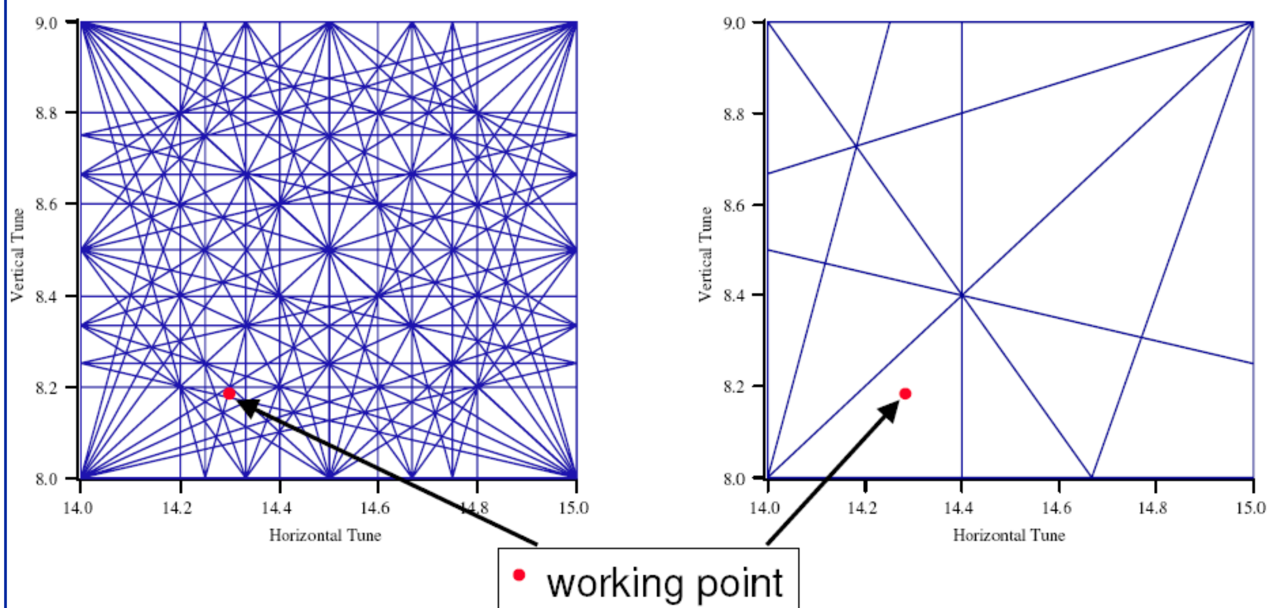
Tune plane:

- Resonances to 5^{th} order
- 12-fold periodicity =x12 resonance reduction
- Only get resonances

$$n\nu_x + m\nu_y = 12 * N$$

- True for periodic magnets (sextupoles), not IDs

All resonances | Allowed resonances



Resonance excitation

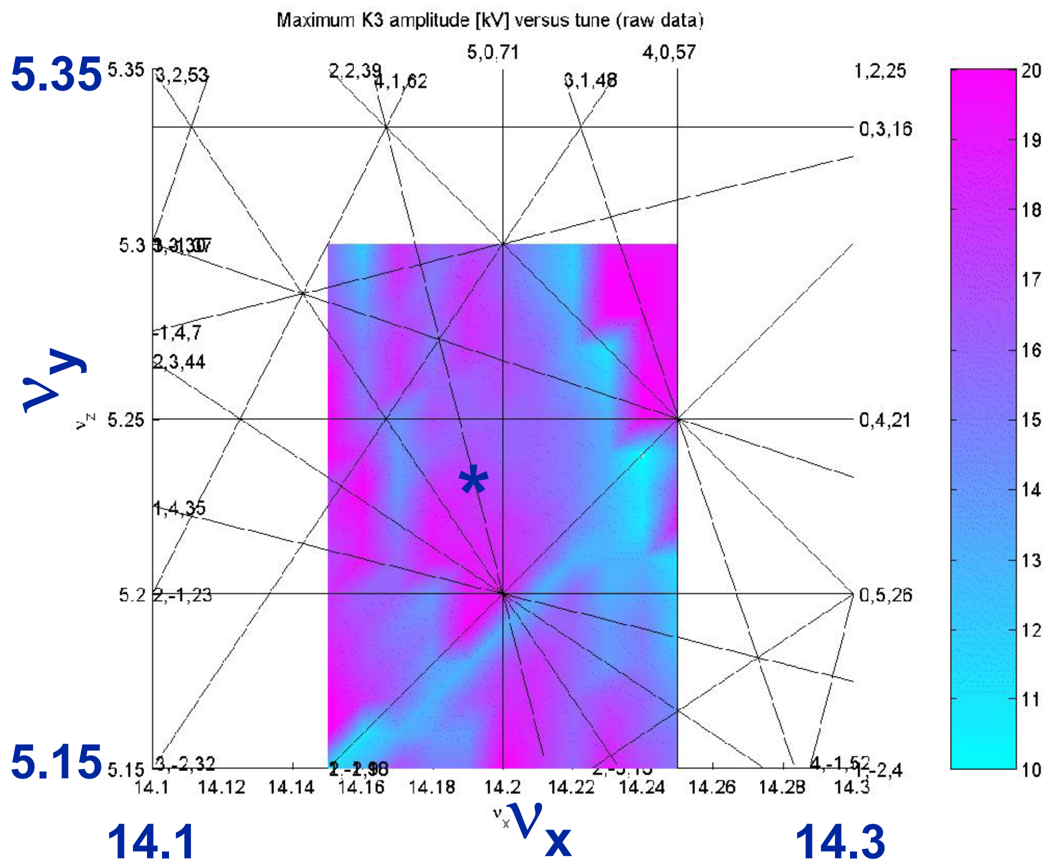
- Resonances can lead to irregular and chaotic behavior in betatron oscillations, which can lead to particle loss.

  Rule of thumb ---- Avoid low order resonances

- Unfortunately there is no simple way to forecast the real strength of resonances without using a tracking code or through measurements
  - Tune scans
  - Frequency map analysis

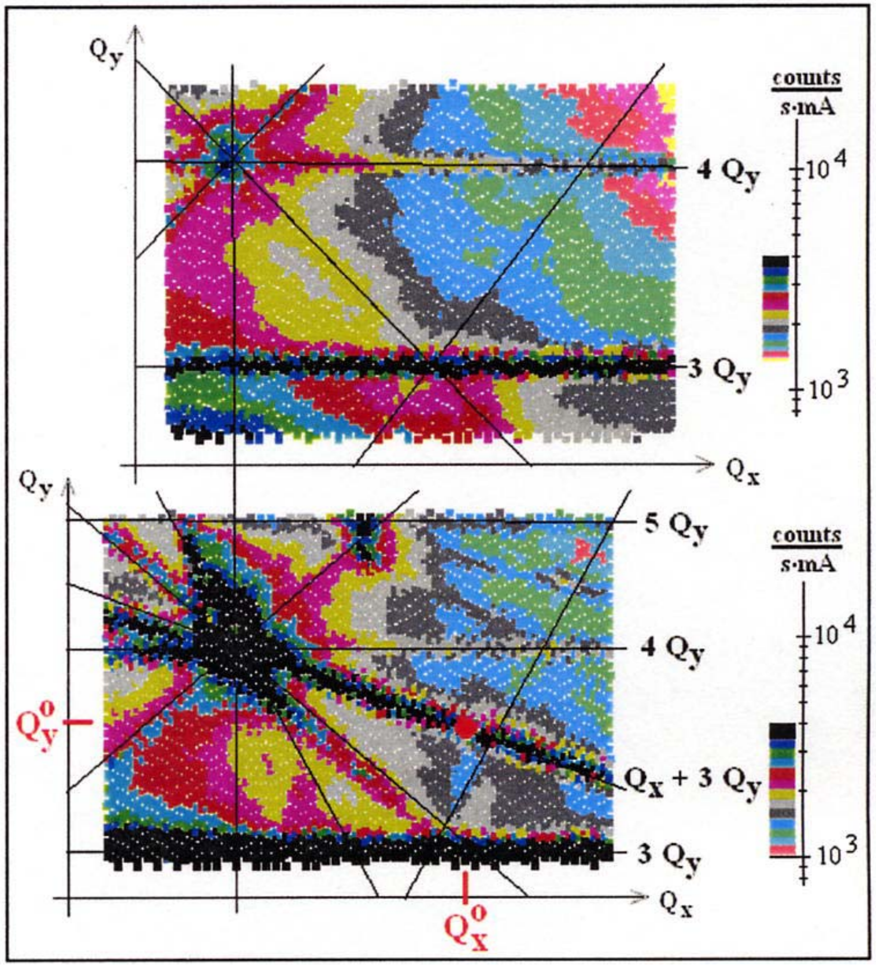
Dynamic aperture vs. tune

- **Resonant lines:**
  - $\nu_x - \nu_y = 9$
  - $3\nu_x + \nu_y = 48$
  - $4\nu_x + \nu_y = 62$
- **Resonances offset from tune shift with amplitude.**
- *** = operating tunes (14.19, 5.23)**
- **Data gathered automatically on owl shift.**



BESSY beam loss monitor measurement

At BESSY, the beam loss was measured as a function of tunes. The additional losses associated with an insertion device showed a problem with nonlinear fields.



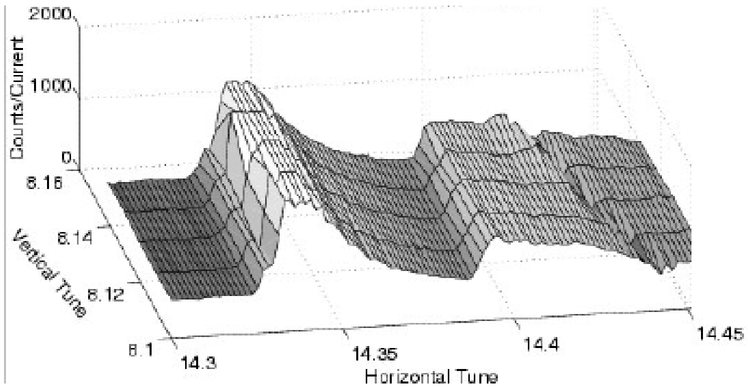
Insertion device open

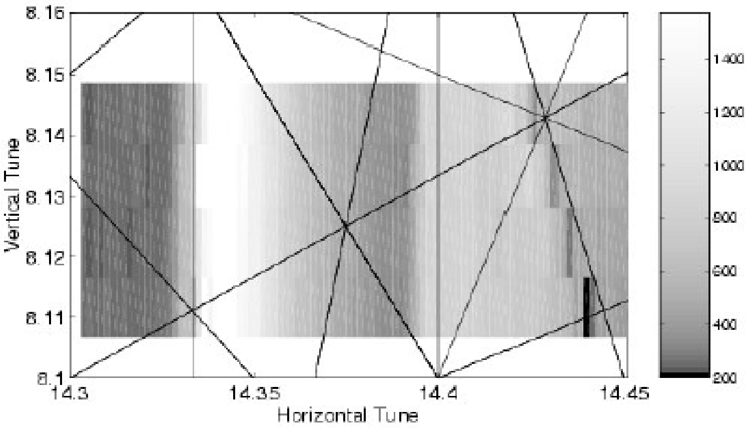
Insertion device closed

Kuske et al., PAC01.

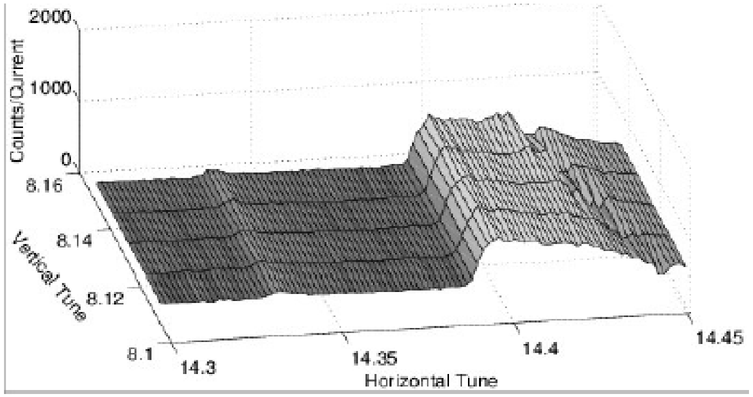
ALS tune scans (with and w/out β-beating)

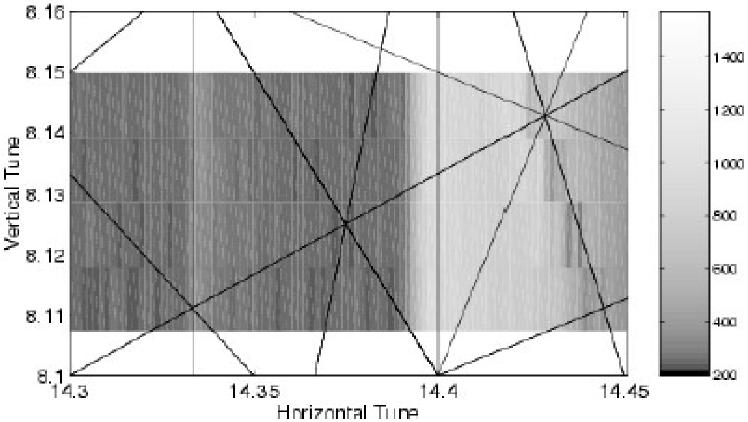
Uncorrected lattice





Corrected lattice





Three resonances are present:

$5\nu_x = 72$ (allowed)

$3\nu_x = 43$ (unallowed)

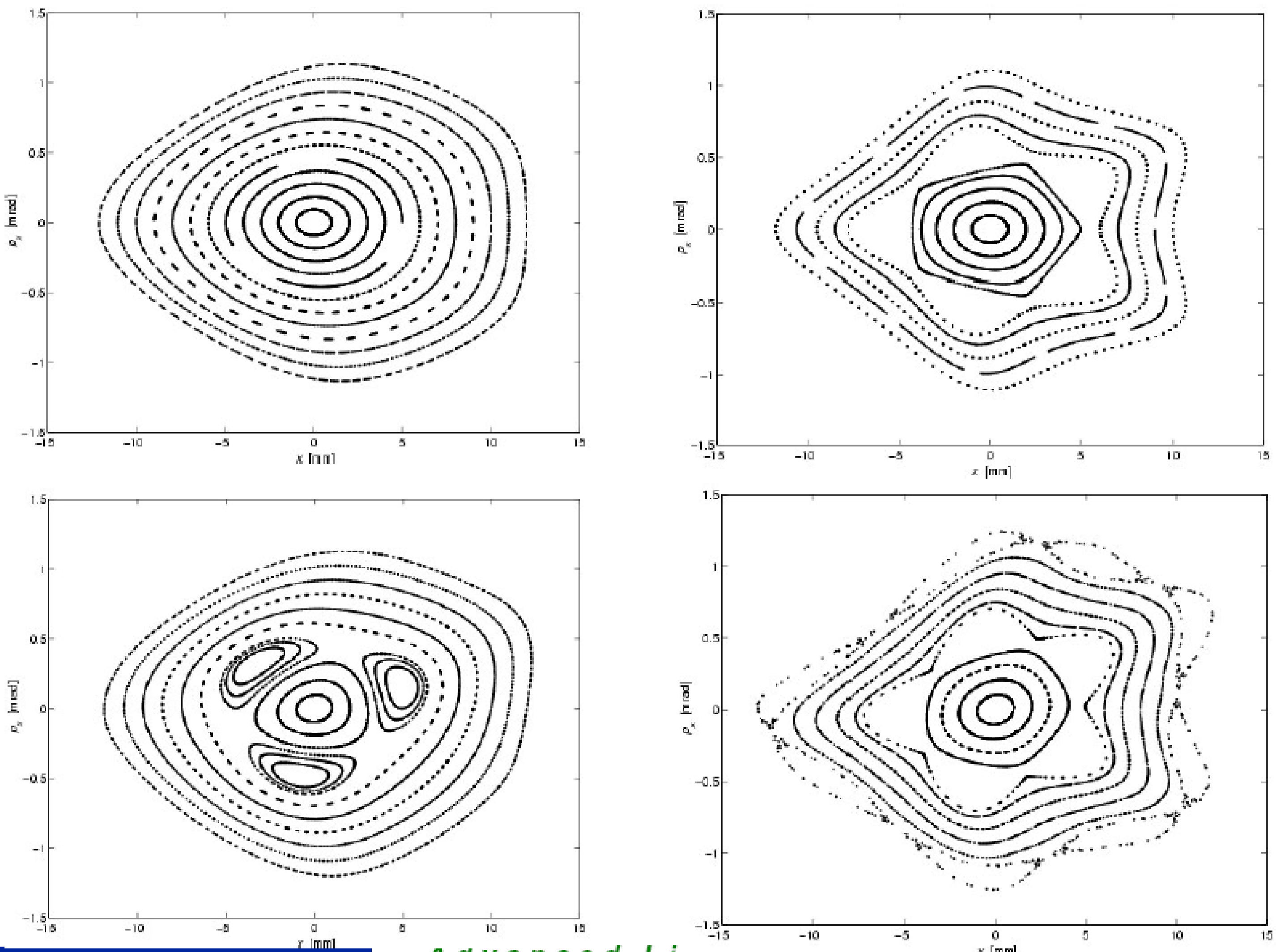
$2\nu_x - \nu_y = 37$ (unallowed)

ALS computer tracking results show benefit of correcting periodicity of linear optics

Near 3rd order resonance | **Near 5th order resonance**

linear optics corrected

linear optics uncorrected



Tune scan summary

Advantages

Quickly and sensitively see excited resonances in the tails and core of the beam as a function of different tunes

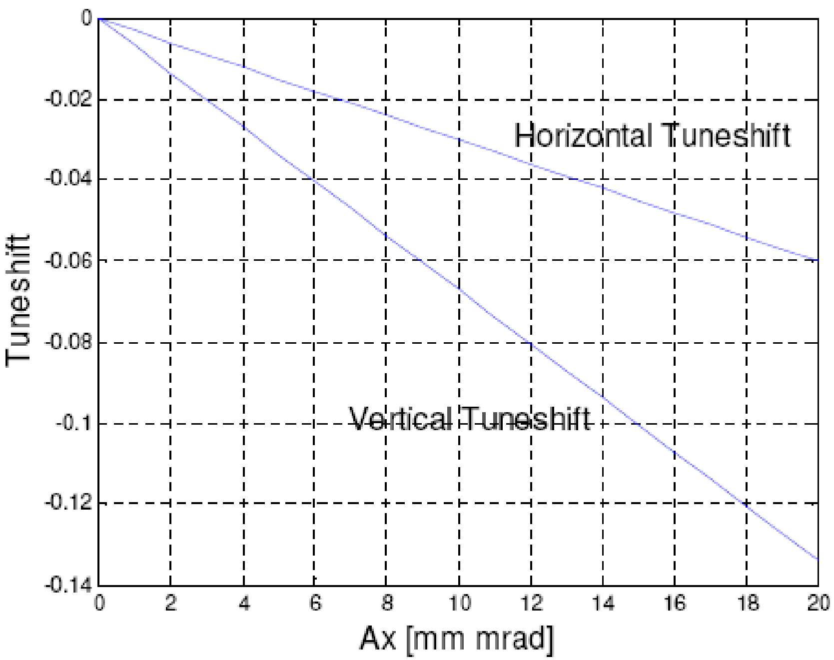
Disadvantages

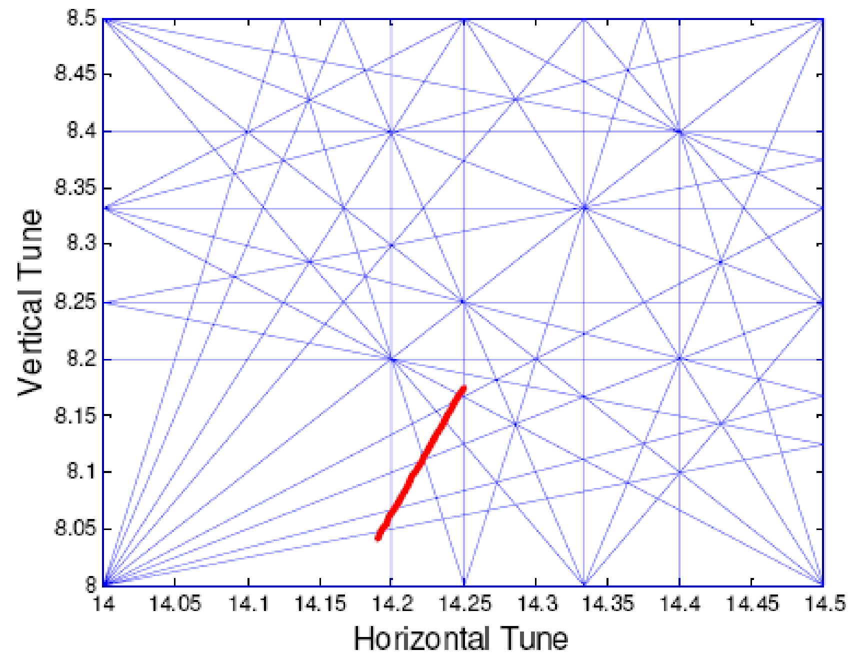
Probing different machines and not looking at the effect of resonances on one working point and at different amplitudes. This is what one really would like to see.

Tune shift with amplitude

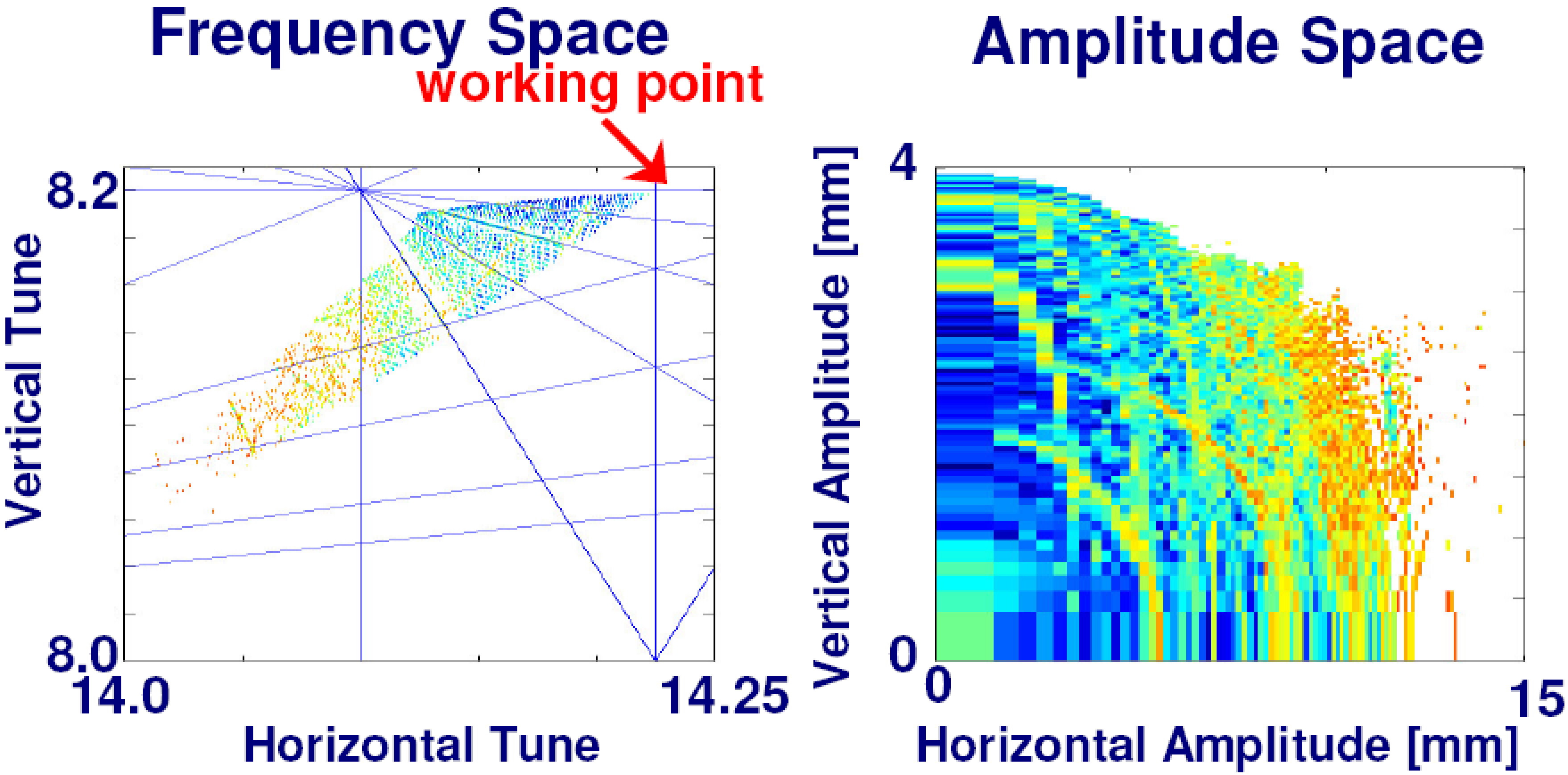
- Electron tunes get shifted with x_β, y_β oscillation amplitude.
- Tune shift with square of amplitude (x_β^2) comes from cubic terms in equation of motion, i.e.

$$x'' + K_x(s)x = A_3 x^3$$



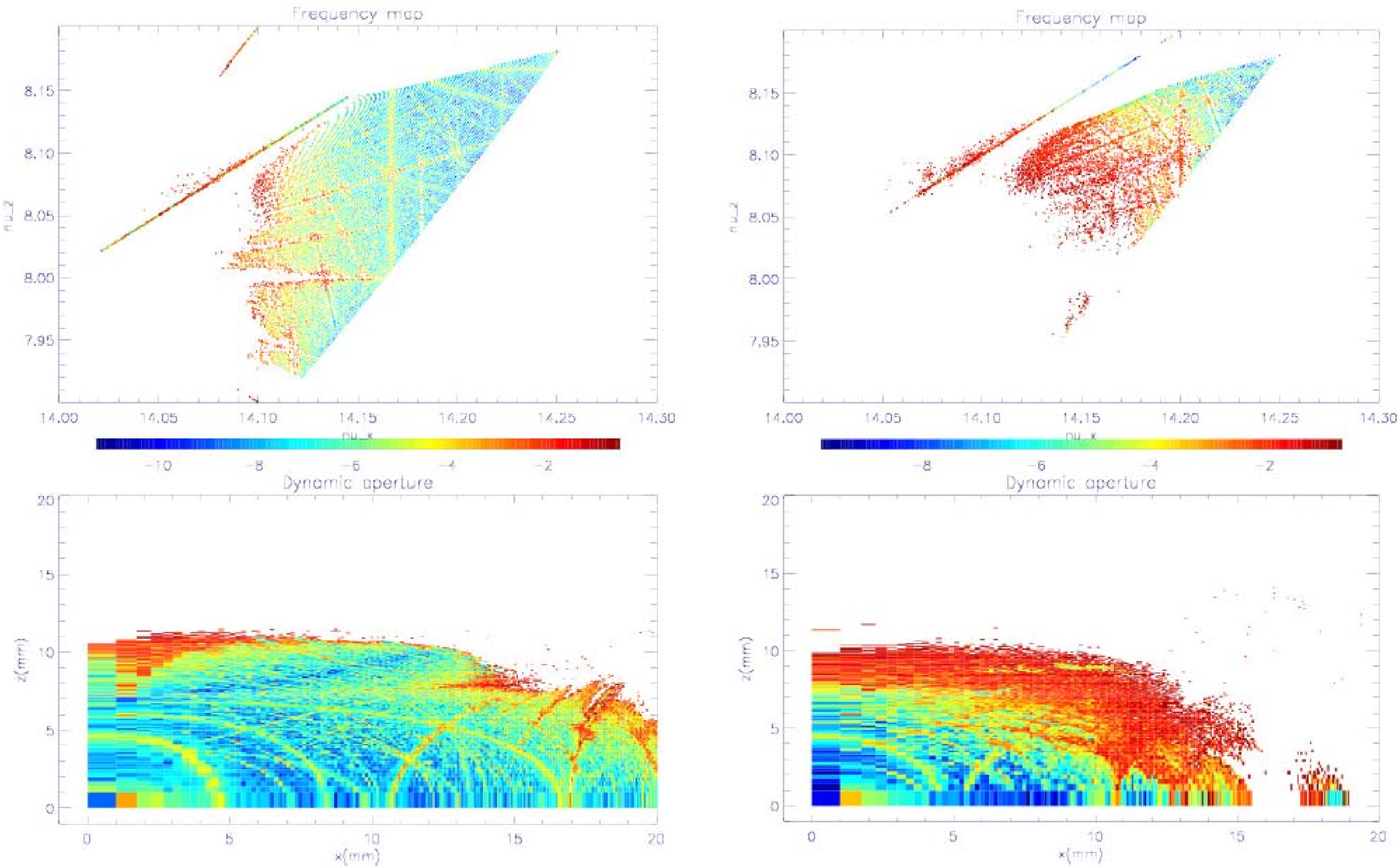


Frequency map analysis



Color scheme refers to diffusion rate of tunes.

Frequency maps – ideal lattice vs. lattice with small linear optics errors.



Frequency maps – experimental set-up

Experimental Hardware

- horizontal + vertical single turn kicker
- 96 turn by turn monitors (1024 turns)

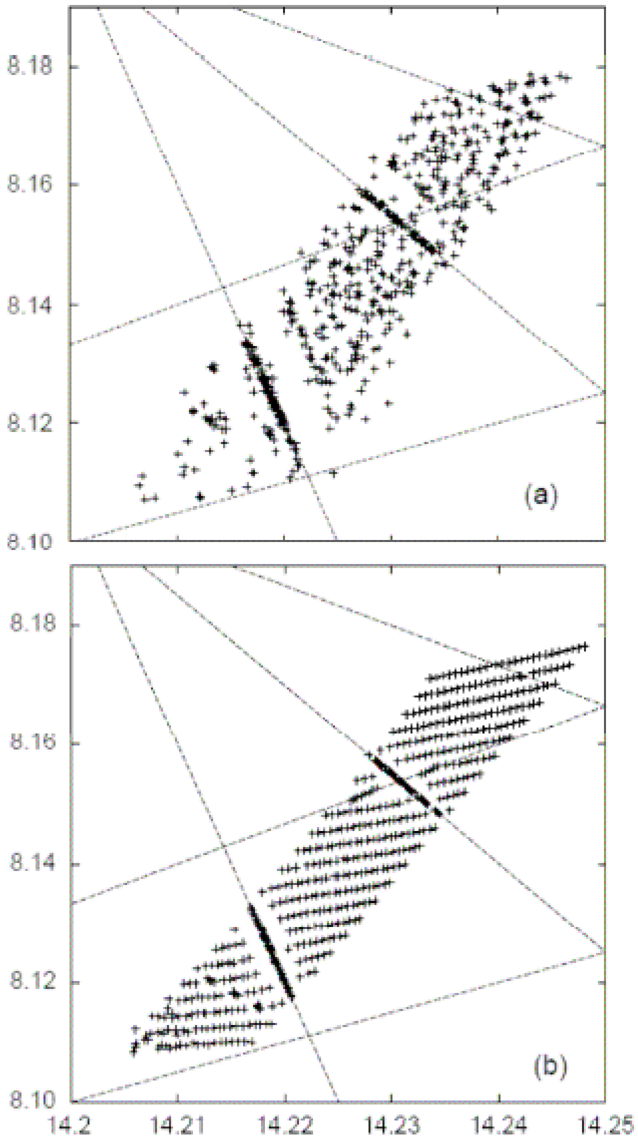
Experimental Procedure

- Electron beam (single bunch or small bunch train) gets simultaneously a horizontal and a vertical kick
- Beam centroid oscillations are recorded turn by turn for 1024 turns
- Repeat with different initial conditions (hor. + vert. kick amplitude) $\rightarrow$ 400-600 total points per map

Data Analysis

- turn by turn data is analyzed with frequency analysis post processor (NAFF) and results plotted in tune plane

Measured frequency maps at ALS



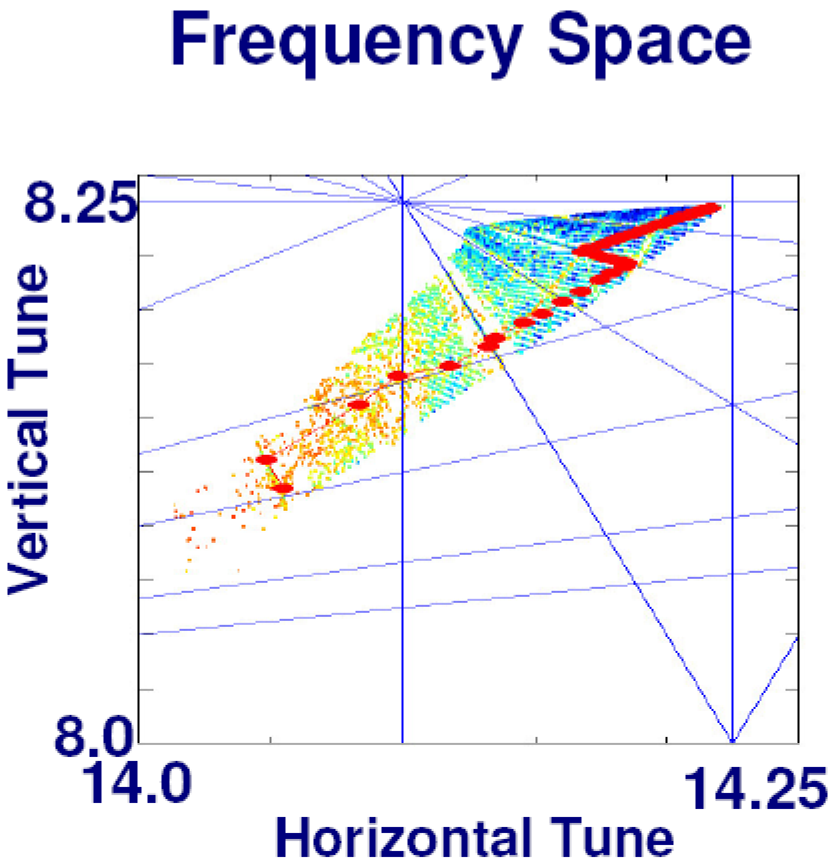
- excellent agreement, using calibrated model (gradient errors), random skew errors, nominal sextupoles

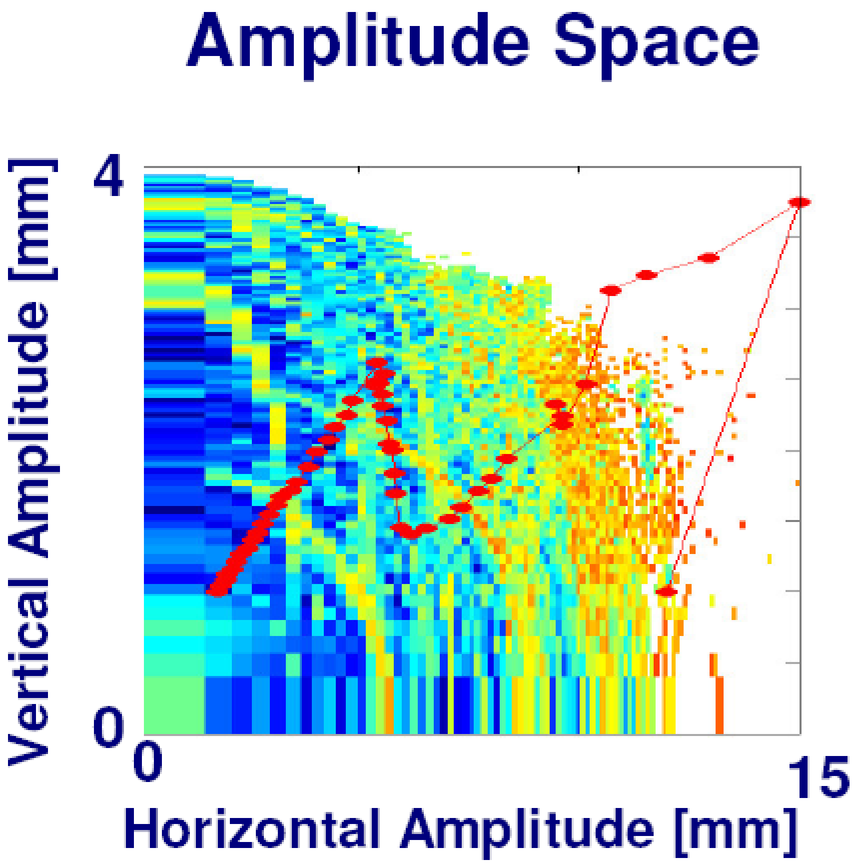
Phys. Rev. Lett. 85, 3, (July 2000), pp.558-561

Also see PAC papers for good measurement results at BESSY-II.

Computer tracking example

Vertical amplitude growth due to resonances:





Measuring nonlinear dynamics - summary

1. Tune scans
2. Tune maps
3. Closed orbit scans (see tomorrows lecture)