Lecture 3 (part 1)
Aspects of Longitudinal Beam Dynamics and Beam Acceleration in Synchrotrons

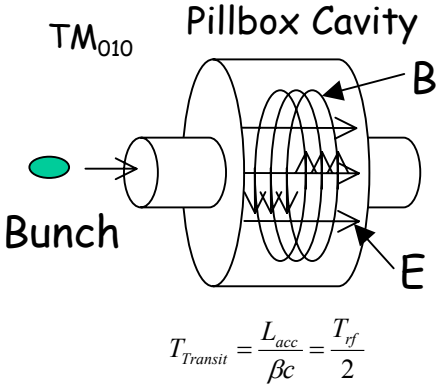
Chandra Bhat

Accelerating RF Cavities

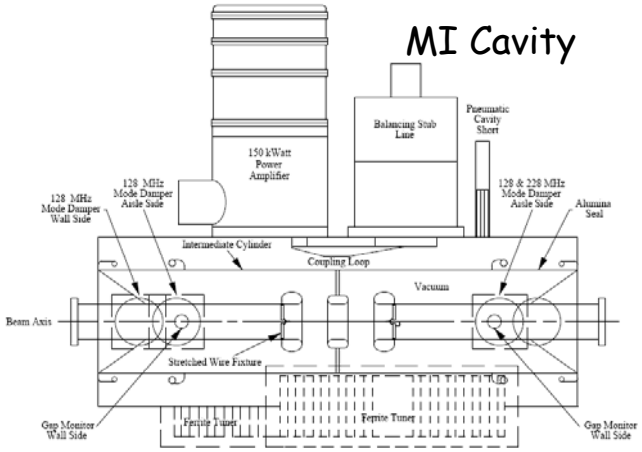
We have seen that most of the charged particle accelerators use accelerating "resonant" rf cavities. For example: cyclotron, synchrocyclotrons, synchrotrons, microtrons and all linear accelerators. The exceptions are betatrons.

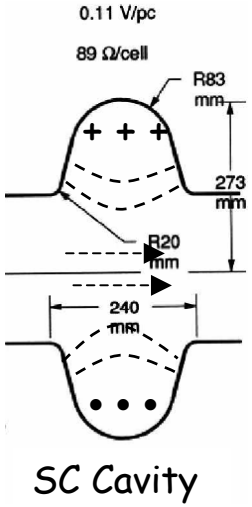
These rf cavities are excited by rf amplifiers. This will not be part of this class. You may learn about these in one of the classes on RF systems.

Examples of rf cavities:



$$T_{Transit} = \frac{L_{acc}}{\beta c} = \frac{T_{rf}}{2}$$



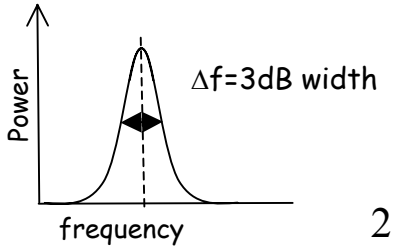


SC Cavity

For fundamentals of electro-magnetic rf cavity resonators refer to Feynman Lectures II Chapter 23.

Generally, fundamental mode of the resonant cavity is used for beam acceleration. The Q value of the cavity at that frequency is given by,

$$Q = 2\pi \frac{\text{Energy Stored in Cavity}}{\text{Energy Lost per Cycle}} = \frac{\Delta f_{rf}}{f_{rf}}$$



Q(MI) ~ 5000, Q(SC)~10^9 etc. If Q=1 then they are called **wideband** rf cavities.

Accelerating RF Cavities (cont.)

In low and medium energy synchrotron accelerators, the frequency of the cavity need to be varied continuously during beam acceleration. For example,

Fermilab Booster (400MeV-8GeV)-	38MHz-52.81MHz
Main Injector (8-150GeV)-	52.81-53.14MHz
Tevatron (150GeV-1TeV)-	53.14MHz→ 53.14MHz +1kHz

Ferrite loaded tuners are commonly used to vary the frequency of the cavity if the frequency range is large . However, in the case of the Tevatron the cavities are tuned to required frequency by varying the temperature of the cooling water.

In any case, the accelerating E-field at the accelerating gap is given by,

$$E = E_0 e^{i\omega t} \quad \text{with } \omega = 2\pi f_{rf} \quad f_{rf} - \text{rf resonant frequency}$$

$$f_{rf} = h f_{rev}$$

h=harmonic number,

f_{rev}= revolution frequency of the particle

Or the accelerating rf voltage is,

$$V = V_0 \sin\phi; \quad \phi = \omega t$$

In a circular accelerator the particles are accelerated by repeated passage through the same rf cavity or a number of rf cavities.

Example: In case of the Tevatron, V_0~1MV and $\phi \approx 10$deg. The beam circulates about 4.77×10^6 times to accelerate from 150 GeV to 1 TeV in about 100 sec.

Equations of Motion and Longitudinal Phase-space

Let the synchronous particle arrive at the accelerating gap at a phase ϕ_s. Then, it always receives an additional energy of,

$$\Delta E = eV_0 \sin(\phi_s);\ V_0 \text{ is peak rf voltage } \&$$

The subscript 's' stands for synchronous particle

1

Now let us investigate the situations for non-synchronous particles.

Let us define variables for any other particle as,

Revolution frequency: $f_r = f_s + \Delta f$

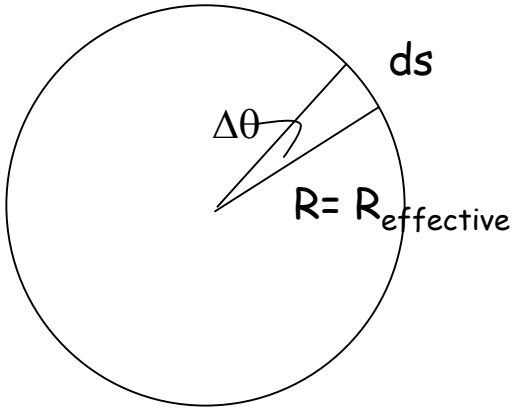
$\omega_r = \omega_s + \Delta\omega$

RF phase : $\phi = \phi_s + \Delta\phi = \phi_s + \psi$

Momentum: $p = p_s + \Delta p$

Energy : $E = E_s + \Delta E$

Azimuthal angle : $\theta = \theta_s + \Delta\theta$



The azimuthal angle $\Delta\theta$ is related to "ds" by,

$$ds = R\,\Delta\theta$$

Over one revolution this angle θ changes by 2π radians, while the rf phase ϕ changes by $2\pi h$. Hence,

$$\psi = \Delta\phi = -h\,\Delta\theta$$

2

The negative sign comes from the fact that a particle behind the synchronous particle arrives later in time. But $\Delta t > 0$.

Equations of Motion & Long. Phase-space (cont.)

The angular velocity ω is given by

$$\omega = \frac{d\theta}{dt} \Rightarrow \Delta\omega = \frac{d\Delta\theta}{dt} = -\frac{1}{h}\frac{d\Delta\phi}{dt} = -\frac{1}{h}\frac{d\psi}{dt} = -\frac{1}{h}\frac{d(\phi-\phi_s)}{dt} = -\frac{1}{h}\frac{d\phi}{dt} \quad \because \ \phi_s \text{ is our reference phase angle.}$$

The revolution period T is given by

$$T = \frac{C}{c\beta} = \frac{1}{f_r} \Rightarrow \frac{\Delta T}{T} = -\frac{\Delta f}{f_r} = \left[\frac{\Delta C}{C} - \frac{\Delta\beta}{\beta}\right] = \left[\frac{\Delta C}{C} - \frac{1}{\gamma^2}\frac{\Delta p}{p_s}\right] = \eta\,\frac{\Delta p}{p_s}\,; \text{ with } \quad \eta = \left[\alpha_c - \frac{1}{\gamma_s^{\,2}}\right] \Rightarrow \left[\frac{1}{\gamma_T^{\,2}} - \frac{1}{\gamma_s^{\,2}}\right]$$

$$\Delta p = -\frac{p_s \Delta f}{\eta f_s} = -\frac{p_s \Delta\omega}{\eta\omega_s} = \frac{p_s}{h\eta\omega_s}\frac{d\phi}{dt}$$

Further, ΔE and Δp are related by,

$$\Delta E = \beta_s c \Delta p = \omega_s R_s \Delta p \ \Rightarrow \quad \frac{d\phi}{dt} = \frac{h\eta}{p_s R_s}\Delta E \qquad \text{3}$$

From Eq. 1 the average momentum gain per turn is

$$(\Delta p)_{turn} = \frac{eV_0}{\omega_s R}\sin\phi$$

Thus the rate of change of momentum is given by

$$\frac{(\Delta p)_{turn}}{T} \approx \frac{dp}{dt} = \dot{p} = \frac{eV_0}{T\omega_s R}\sin\phi = \frac{eV_0}{2\pi R}\sin\phi$$

or $\quad 2\pi(R\,\dot{p}) = eV_0\sin\phi \qquad$ 4

Equations of Motion & Long. Phase-space (cont.)

For the synchronous particle we get,

$$2\pi(R\dot{p})_s = eV_0 \sin\phi_s \qquad \text{5}$$

Taking the difference between Eqs. 4 and 5 we get,

$$2\pi\,\Delta(R\dot{p}) = eV_0(\sin\phi - \sin\phi_s) \qquad \text{6}$$

Now $\Delta(Rp)$ can be written as,

$$\Delta(R\dot{p}) = R\dot{p} - R_s\dot{p}_s = (R_s + \Delta R)(\dot{p}_s + \Delta\dot{p}_s) - R_s\dot{p}_s$$

$$\approx \Delta R\,\dot{p}_s + R_s\Delta\dot{p}_s = \Delta R\left[\frac{dp}{dt}\right]_s + R_s\Delta\dot{p}_s \approx \left[\frac{dR}{dt}\right]_s \Delta p + R_s\Delta\dot{p}_s = \frac{d}{dt}(R_s\Delta p)$$

$$\therefore \quad 2\pi\Delta(R\dot{p}) = 2\pi\frac{d}{dt}(R_s\Delta p) = 2\pi\frac{d}{dt}\left[R_s\frac{\Delta E}{\omega_s R_s}\right] = 2\pi\frac{d}{dt}\left[\frac{\Delta E}{\omega_s}\right] \qquad \text{7}$$

This comes from Eq. 3

Thus, the equation of motion in (ΔE, ϕ)-phase space is given by,

$$\frac{d(\Delta E)}{dt} = \frac{\omega_s}{2\pi}eV_0(\sin\phi - \sin\phi_s)$$

$$\frac{d\phi}{dt} = \frac{h\eta}{p_s R_s}\Delta E = \frac{h\eta\omega_s^{\,2}}{\beta_s^{\,2}E_s}\left[\frac{\Delta E}{\omega_s}\right] \qquad \text{8}$$

Synchrotron Oscillation frequency and Transition Jump

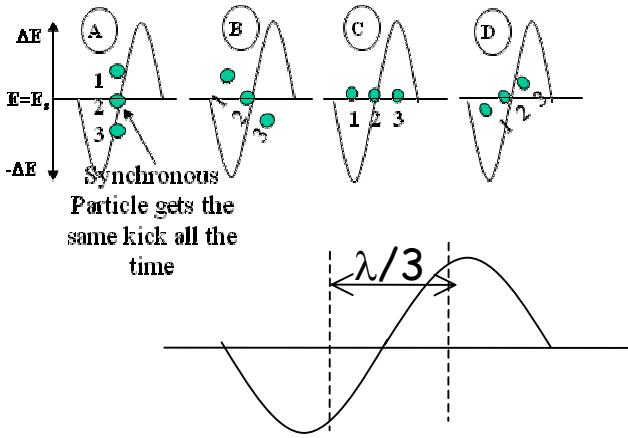
Eliminating ΔE in Eq. 8 we get,

$$\frac{d}{dt}\left[\frac{p_s R_s}{h\eta\omega_s}\frac{d\phi}{dt}\right]+\frac{1}{2\pi}eV_0(\sin\phi-\sin\phi_s)=0 \qquad (9)$$

Assume that R_s, p_s, η, ω_s and V_0 are all constant (or vary very slowly with respect to time. Besides, $\Delta\phi=\phi-\phi_s \ll 1$ then the Eq. 9 becomes

$$\frac{d^2\phi}{dt^2}+\frac{eV_0 h\eta\omega_s\cos\phi_s}{2\pi\ p_s R_s}\phi=0$$

or $$\frac{d^2\phi}{dt^2}+\Omega_s{}^2\phi=0;\quad \Omega_s=\sqrt{\frac{eV_0 h\eta\omega_s\cos\phi_s}{2\pi\ p_s R_s}} \qquad (10)$$



The quantity Ω_s is called "synchrotron oscillation frequency". For stable longitudinal motion the synchrotron oscillation frequency must be a real quantity.

Synchrotron Tune: $\nu_s=\Omega_s/\omega_s$

Consequently, during beam acceleration $\eta\cos\phi_s>0$. This implies,

$$\gamma<\gamma_T \quad \eta>0 \quad \therefore \quad 0<\phi_s<\frac{\pi}{2}$$

$$\gamma>\gamma_T \quad \eta<0 \quad \therefore \quad \frac{\pi}{2}<\phi_s<\pi$$

because $\eta=\frac{1}{\gamma^2}-\frac{1}{\gamma_T^2}$

Hence, during the RF acceleration in circular accelerators we must jump the accelerating phase angle $\phi_s \rightarrow \pi-\phi_s$, around the transition energy. This is called "transition phase jump". This is critical for all hadron synchrotron accelerators.

Hamiltonian Formalism and Phase-space Ellipse

The synchrotron equations of motion Eq. 8 can be derived from a "Hamiltonian",

$$H = \frac{h\eta}{2p_s R_s}\Delta E^2 + \frac{\omega_s}{2\pi} eV_0[\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s] \qquad 11$$

From this the equations of motion will be,

$$\frac{\partial H}{\partial \phi} = -\frac{d(\Delta E)}{dt} = -\frac{\omega_s}{2\pi} eV_0(\sin\phi - \sin\phi_s)$$

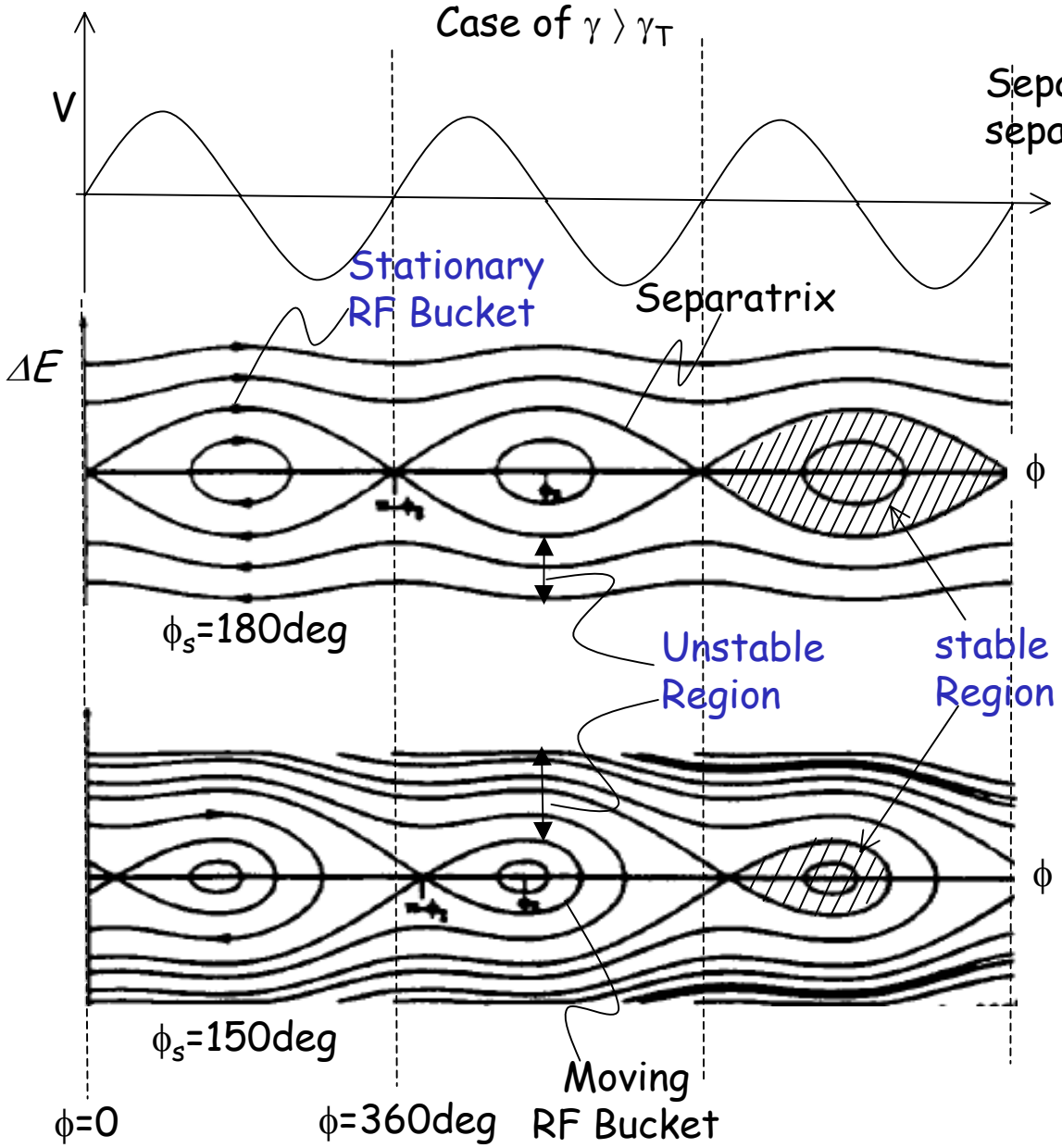
$$\frac{\partial H}{\partial W} = \frac{d\phi}{dt} = \frac{h\eta}{p_s R_s}\Delta E$$

Then the longitudinal motion of the particles can be conveniently represented by their trajectories in (ΔE, ϕ)-phase space . These trajectories will be contours of constant Hamiltonian given by , Eq. 11. Thus, with,

$$H = A\Delta E^2 + B[\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s]$$

$$\Delta E = \pm\sqrt{\frac{H}{A} - \frac{B}{A}[\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s]} \qquad 12$$

Phase-space Ellipse and RF Bucket



Separatrix: the phase space contour separating stable and unstable region

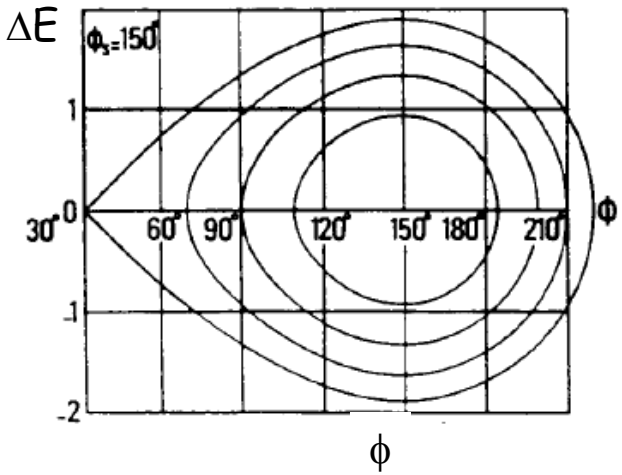


Fig. 6 Stable phase space trajectories

RF Bucket (cont.)

Bucket Area BA: The area enclosed by the separatrix and is given by, $BA = \oint \Delta E(\phi) d\phi$

To calculate the bucket area we derive an expression for the separatrix. We know that the separatrix pass through ($\Delta E=0$, π- ϕ_s). Then,

$$H_{sepx} = 0 + \frac{\omega_s}{2\pi} eV_0[\cos(\pi - \phi_s) - \cos\phi_s + (\pi - \phi_s - \phi_s)\sin\phi_s] = \frac{\omega_s}{2\pi} eV_0[-2\cos(\phi_s) + (\pi - 2\phi_s)\sin\phi_s]$$

The trajectory equation for the separatrix is, $H = H_{sepx}$ and hence,

$$\frac{h\eta}{2p_s R_s}\Delta E^2 + \frac{\omega_s}{2\pi} eV_0[\cos\phi + \cos\phi_s - (\pi - \phi - \phi_s)\sin\phi_s] = 0 \qquad (13)$$

The separatrix has two turning points at ϕ_u and π- ϕ_s points, where $\Delta E=0$. For ϕ_s =0, the turning points are, -π and π for $\gamma < \gamma_T$.

Now we can calculate the bucket area. After some mathematical adjustments we get,

$$BA = 16\sqrt{\frac{p_s R_s \omega_s eV_0}{2\pi h|\eta|}}\alpha(\phi_s)$$

$$\text{where, } \alpha(\phi_s) = \frac{1}{4\sqrt{2}} \int_{\phi_u}^{\pi-\phi_s} \left\{ -\frac{|\eta|}{\eta}[\cos(\phi) + \cos(\phi_s) - (\pi - \phi - \phi_s)\sin(\phi_s)] \right\}^{\frac{1}{2}} d\phi \qquad (14)$$

BA is maximum at $\alpha(\phi_s = 0) = 1$ for $\gamma < \gamma_T$. Such buckets are called "stationary buckets". Similarly for $\gamma > \gamma_T$, $\alpha(\phi_s = \pi) = 1$.

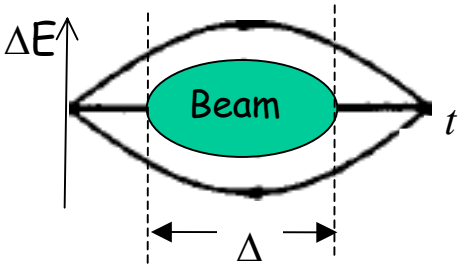
RF Bucket (cont.) and Longitudinal Emittance

Bucket Height: This is the maximum height of a rf bucket, given by,

$$\frac{dp}{p} = \pm\sqrt{\frac{2eV_0}{\pi\beta_s^{\,2}E_s h|\eta|}}\,Y(\phi_s) \quad \text{with } Y(\phi_s) = \sqrt{\left|\cos(\phi_s) - \frac{\pi - 2\phi_s}{2}\sin(\phi_s)\right|}\,;\ \phi_s = 0 \text{ stationary bucket} \qquad (15)$$

Longitudinal Emittance (LE): The phase space area occupied by the beam particles in (ΔE, ϕ)-phase space.

Units of LE= eV-sec



Measurement of LE and Beam Height at Constant Energy:
By measuring bunch length Δ (radian) and rf voltage one can measure the LE of the beam as follows,

$$LE = \sqrt{32}\,\frac{V_0 R_s^{\,2} E_s}{2\pi h^3 c^2 |\eta|}\int_0^{\frac{\Delta}{2}}\sqrt{\cos(x) - \cos\left[\frac{\Delta}{2}\right]}\,dx \quad \text{and} \quad \text{"Beam Height, } \hat{\Delta}\text{E"} = \text{Bucket height} \times \sin\left[\frac{\Delta}{4}\right] \qquad (16)$$

Comments: Measurement of LE of a bunch in moving bucket is quite complicated. This needs precise measurements of ϕ_s.

Beam Transfer and Longitudinal Matching: It is essential that in transferring a bunch from Ring-1 to Ring-2, we should match the bucket heights, i.e.,

$$\left|\frac{V\cos(\phi_s)}{h\eta}\right|_{Ring-1} = \left|\frac{V\cos(\phi_s)}{h\eta}\right|_{Ring-2} \qquad (17)$$

Beam Acceleration

Beam Acceleration: After the beam is captured in matched stationary rf buckets the synchronous phase is changed in sync with the momentum ramp to accelerate the beam. The condition to be met is,

Momentum ramp $\frac{dp}{dt} = eV \sin(\phi_s)$ Matched rf voltage and phase with BA/LE ≈ 2.6 — 18

The synchronous angle must satisfy

$$0 < \phi_s < \frac{\pi}{2} \quad \text{for } \gamma < \gamma_T \text{ and } \frac{\pi}{2} < \phi_s < \quad \text{for } \gamma > \gamma_T$$ — 19

otherwise the net acceleration force cancel out due to synchrotron motion.

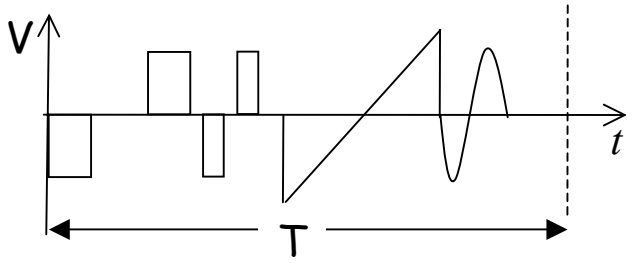
Barrier Buckets

So far we have talked about rf buckets formed by sinusoidal rf waves generated by resonant rf cavities. About 25-years ago a new type of rf system called "barrier" or "wide-band" rf systems were developed at Fermilab by Jim Griffin. These systems can produce arbitrary voltage waveforms and have been adopted in accelerator operation at various places in the world.

The Recycler at Fermilab uses a barrier rf system exclusively for all of its rf manipulations. A set of solid-state power amplifiers are used to energize these cavities.

The Hamiltonian for an arbitrary barrier wave form is given by,

$$H = -\frac{\eta}{2\beta_s E_s}\Delta E^2 - \frac{1}{T_0}\int_0^t eV(t)dt \qquad (20)$$



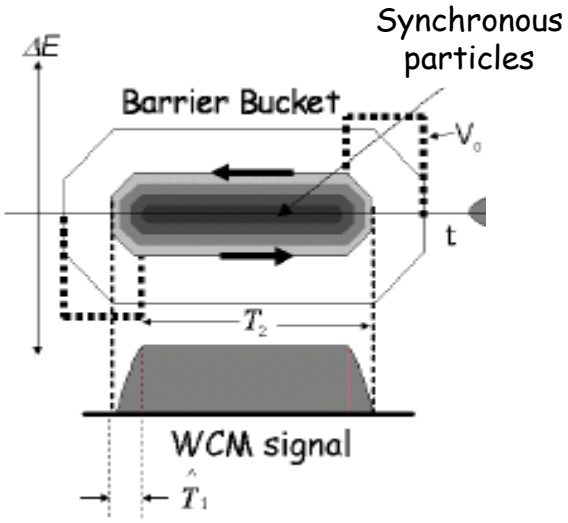
For beam in rectangular bucket the $\hat{\Delta E}$ and LE are given by,

$$\hat{\Delta E} = \sqrt{\frac{2\beta_s^2 E_s}{|\eta|}\frac{eV_{rf}T_1}{T_0}} \qquad (21)$$

$$LE = 2T_2\hat{\Delta E} + \frac{8\pi|\eta|}{3\omega_s\beta_s^2 E_s eV_0}\hat{\Delta E}^3 \qquad (22)$$

$T_2 =$ Gap between rectangular barrier

$T_1 =$ Extent of beam penetration into the barrier



Simulation of Longitudinal Beam Dynamics

There are a number of longitudinal beam dynamics simulation codes available for accelerator design and applications. For example:

1. ESME- Jim. MacLachlan, Fermilab, http://www-ap.fnal.gov/ESME/
   This code is widely distributed and is available on web
2. TIBETAN – G. Wei, from BNL
3. LONG1D – Shane K.
4. ACCEL- John G., Los Alamos National Laboratory

ESME uses the difference equations

$$E_{i,k} = E_{i,k-1} + e[V(\phi_{s,k} + h\vartheta_{i,k}) - V(\phi_{s,k})]$$

$$\phi_{i,k} - \phi_{i,s} = \vartheta_{i,k} = \frac{\tau_{s,k-1}}{\tau_{s,k}}\vartheta_{i,k-1} + 2\pi\left(\frac{\tau_{i,k}}{\tau_{s,k}} - 1\right) \qquad \text{with} \quad \frac{\tau_{i,k}}{\tau_{s,k}} \approx \left[1 + \eta\beta_s^2 \frac{\Delta E}{E_s}\right]_k$$

The subscripts and label the quantities related to particle and to synchronous particles, respectively. The *k*-th energy increment is at the end of the *k*-th turn.

This is a 2D program. Will handle space-charge effects, beam-loading compensation, cavity impedances, beam pipe effects etc.

Lecture 3 (part 2)
Beam Diagnostics

Chandra Bhat

Beam Diagnostics

Beam diagnostics is an essential constituent of any accelerator. Without proper diagnostics we are blind. **"An accelerator is just as good as its diagnostics."**

Beam diagnostic devices can be classified broadly into two types

a.Destructive
b.Non-destructive

The usage of these depends on what we want to accomplish.

Signal processing, associated electronics, analogue signal or digital data treatments, are also subjects of great importance. You will hear from Dr. Okugi.

Below table (H. Koziol, CAS94-01) gives a general list of various devices commonly used in accelerators.

Diagnostics Devices & Beam Properties Measured

PROPERTY MEASURED →	Intensity I , Q	transverse: Position	transverse: Size/shape	transverse: Emittance	longit.: Size/shape	longit.: Emittance	Q-value + ΔQ	Energy + ΔE	Polarization	Effect on beam: N	–	+	D
Beam transformers	●				●	●				x			
Wall-current monitors	●	●			●	●				x			
Pick-ups	●	●	(●)		●	●				x			
Faraday cup	●												x
Secondary emission monitors	●	●	●	●				•			x	x	
Wire scanners		●	●	●				•			x		
Wire chamber		●	●								x	x	
Ionization chamber	●										x	x	
Beam loss monitors		•	•	•			•			x			
Gas curtain/jet		●	●	●							x		
Residual gas monitors		●	●	●						x			
Scintillator screens		●	●								x	x	x
Scrapers, targets		●	●	●									x
Schottky scan	●			●		•	●	●		x			
Synchrotron radiation		●	●		●	●				x			
LASER-Compton scattering			●	●					●	x			
Q-measurement							●			x	x		
Emittance measurement				●							x	x	x
Measurement of energy								●		x	x	x	x
Polarimeter									●	x			x

Effect on beam

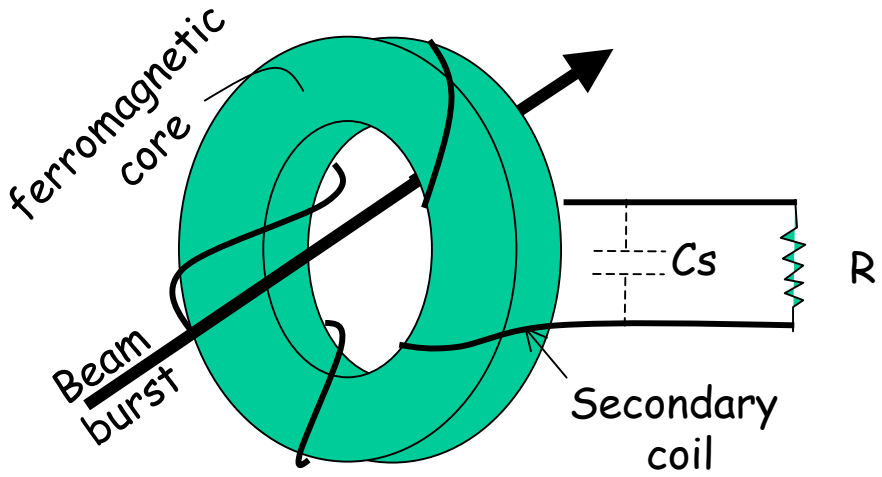
- N none
- – slight, negligible
- + perturbing
- D destructive

Next, we will touch upon basic principles of some of these.

Beam Intensity Monitors

Beam Current Transformers:

This device measures the total number of electric charges in a burst of charged particle beam.



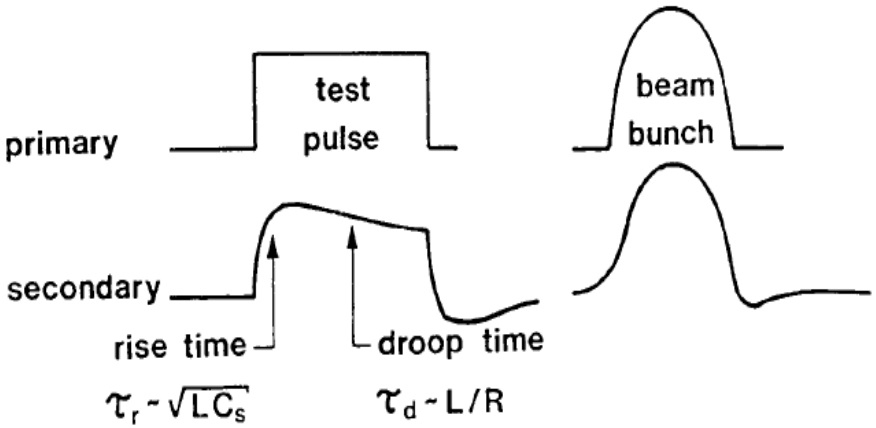


Fig. 4 Signal from real transformer.

Principle: This consists of a ferromagnetic core with a high permeability metal tape wound as shown here. The beam is made to go through as shown in the figure.

A burst of beam acts as a single turn primary winding and the induced voltage in the secondary is given by,

$$V = L\frac{dI_b}{dt}$$

By terminating the secondary using a resistor one can measure the induced voltage- which is proportional to the beam current.

The band width of these devices are generally ~100MHz.

Beam Intensity Monitors (cont.)

Direct Current Transformer:

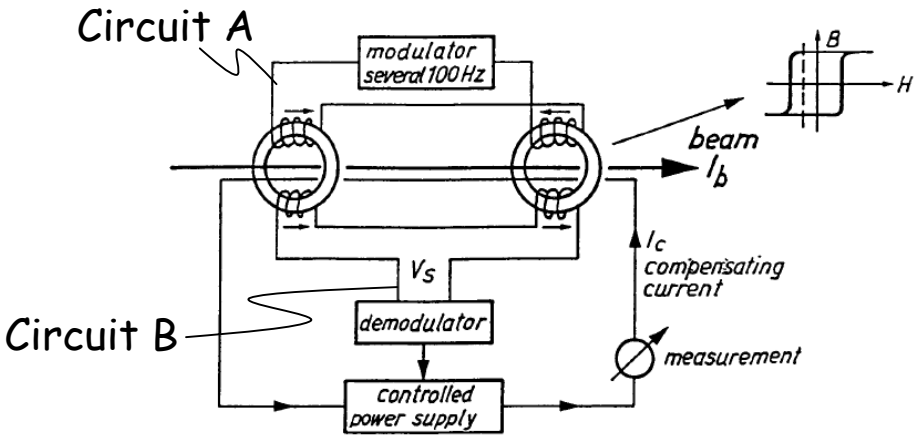
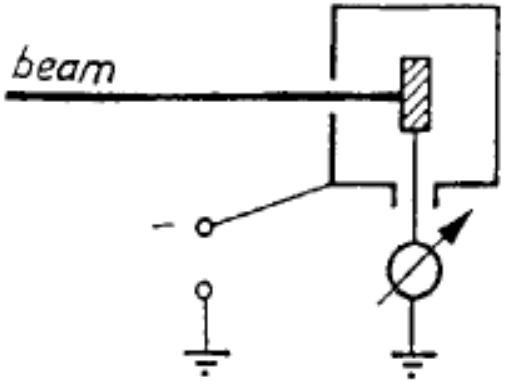


Fig. 7 Basic scheme of a dc beam transformer and rectangular hysteresis of core material.

Principle: In the absence of particle beam through the two ferrite rings, F1 and F2, the induced voltage Vs in "Circuit-B" due to an AC (~100Hz) in "Circuit-A" will be zero. A charged particle beam through the ferrites **introduces bias in the excitation of the cores**. Then, Vs $\neq$ 0., which in turn changes the compensating current. This effect can be used to measure the DC beam intensity. With an accuracy better than 2%.

Faraday Cups:

This is a beam stopper and is the earliest device used to measure the total amount of charges in the beam. Use of this is a destructive technique.



Faraday cup.

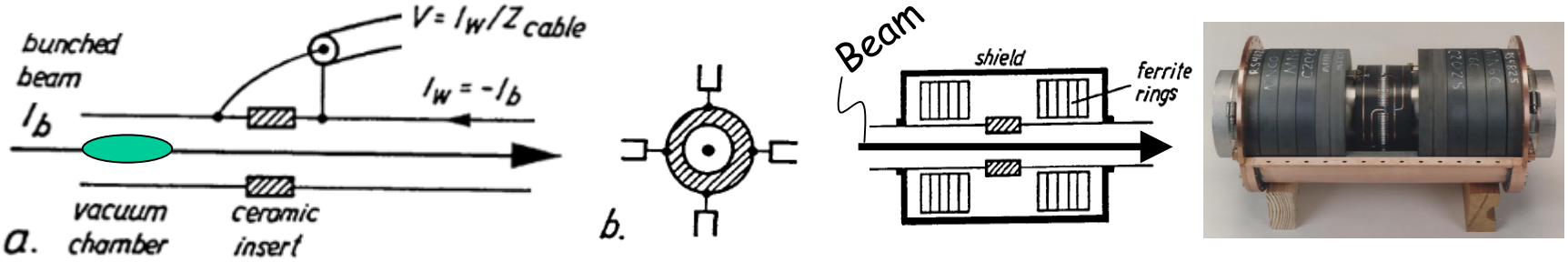
Wall Current Monitors (WCM)

This device is primarily used to measure the **line-charge distribution** of beam bunches in an accelerator or beam-line. This can also be used as an intensity monitor.

Principle: A charged particle beam , I_b in an accelerator is always accompanied by a "wall current", I_w on its beam pipe due to its image charge, so that,

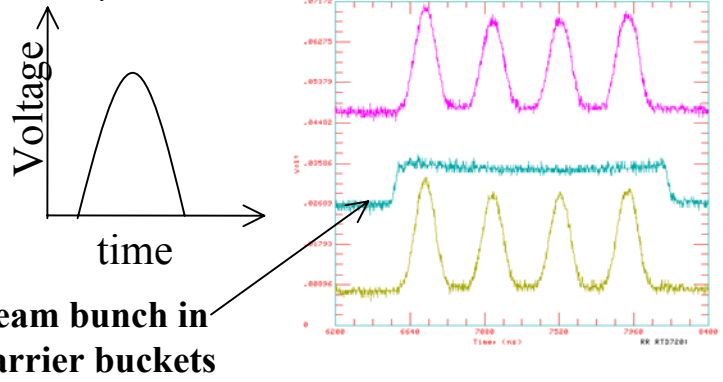
$$I_w = -I_b$$

A gap bridged with resistors can be used to measure this wall current.

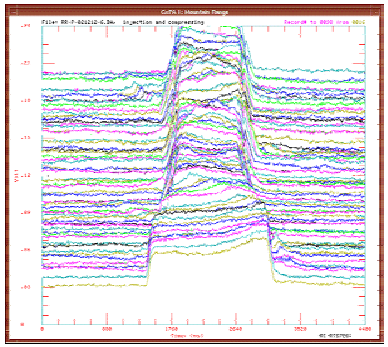


a) Principle of wall-current monitor. b) Separate pick-up of signals to observe beam position.

Examples:



Mountain Range



WCM as Intensity Monitors:

By integrating the area under each bunch one can measure the total charge/bunch. Individual bunch intensity in the Tevatron and in the MI are measured adopting this technique.

Beam Position Monitors (BPM)

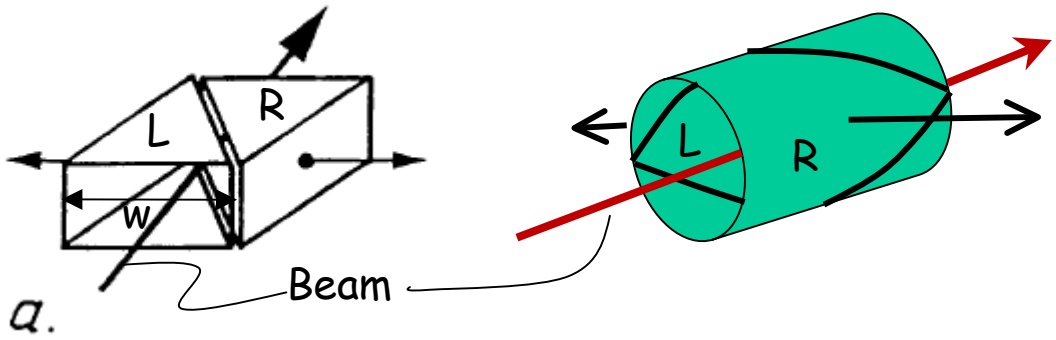
This is used to measure the **transverse beam position** in a beamline and accelerators. The beam has to be bunched.

These devices are available in three types,

1.electrostatic,
2.Magnetic
3.electromagnetic

Widely used in circular accelerators and beamlines

The simplest form of the electrostatic BPM is a rectangular (or cylindrical) box cut diagonally. For example,



MI Cylindrical type BPM

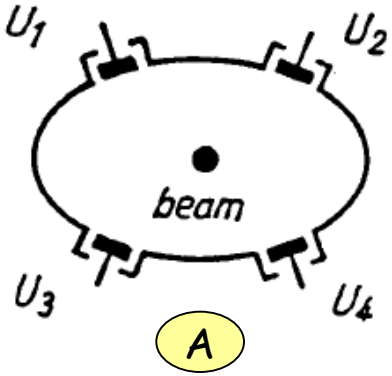
Principle: As the beam passes through the BPM it will induce more electric charges on the metallic electrode closer to the beam than the other. Then,

$$x = \frac{\mathrm{w}}{2}\frac{U_R - U_L}{U_R + U_L};\ \mathrm{w} = \text{width of the BPM}$$

These type of BPMs, properly installed, can be used to measure the beam position either in horizontal or vertical plane in the ring.

Beam Position Monitors (cont.)

Simultaneous measurements of H and V coordinates can be done using special BPMs like,



MI BPM inside a Quad

These types of BPMs are used in the MI and buried in the MI quadrupoles.

With digital electronics, we can measure the beam positions to an accuracy better than fraction of mm. For linear collider at collision points one need better resolution.

In medium energy synchrotrons where rf frequency changes in a large range, the BPMs are used to control the beam acceleration.

In e+ and e- machines, BPMs with "button " electrodes are used. The sectional view of these detectors look similar to that shown in Figure A

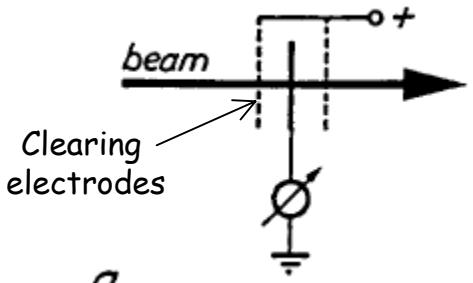
Beam Profile Monitors

These devices are used to measure horizontal and vertical beam profiles in an accelerator or a beam transport system. This technique is semi-destructive.

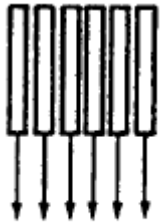
Secondary Emission Monitors (SEM):

Principle: This uses the principle that the electrons are liberated from the surface under the impact of charged particle beam, thus producing a flow of current.

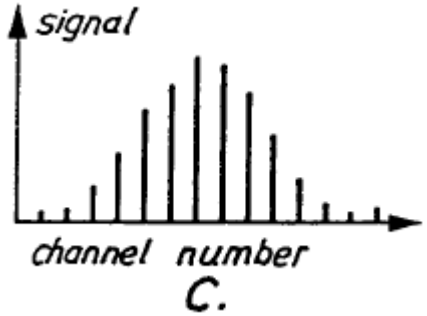
This type of beam profile monitor consists of an array of thin ribbon of special materials with high secondary electron emission coefficient.



Sideway view

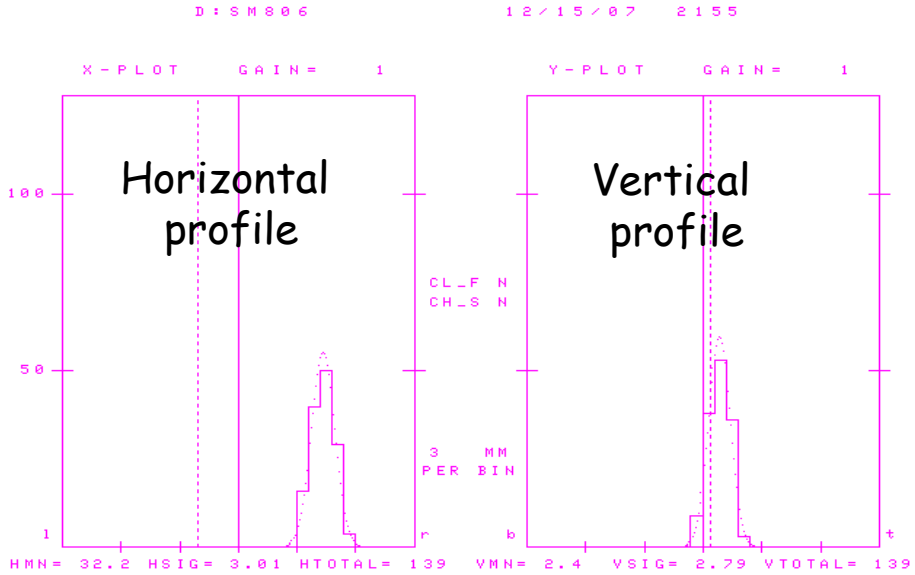


Front view



Profile

Data from Debuncher to Accumulator beamline

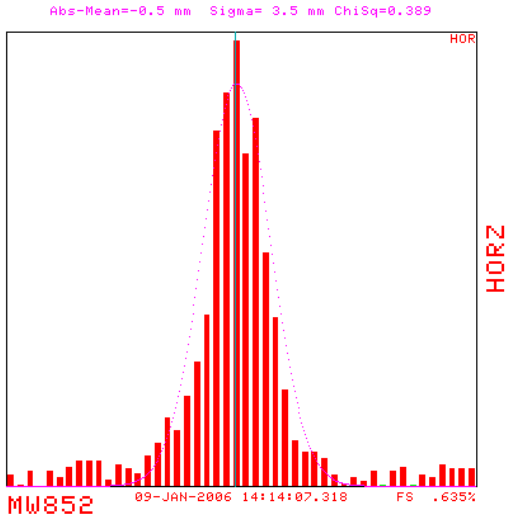


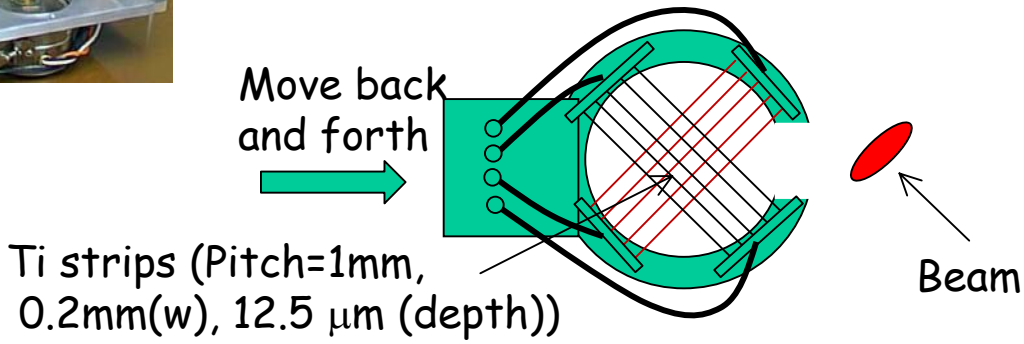
This device is quite sensitive to very low intensity beam. For example in the above figure I~1E7 pbars

Beam Profile Monitors (cont.)

Multi-wire Chambers (MW):
Principle: This uses the principle that the electrons are liberated under the impact of charged particle beam, thus producing a flow of current. Difference between the SEM and MW is that the latter is not a surface phenomenon.

Many of the Fermilab MWs in MI use gold plated tungsten wires (dia. ~75μm) with 1mm pitch.

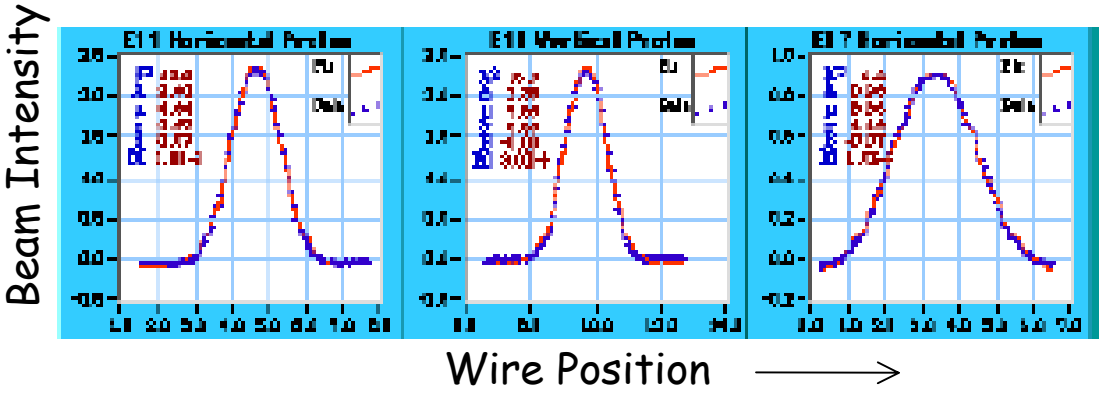




Beam Profile Monitors (cont.)

Wire Scanners:
Principle: Instead of multiple wires as in the case of SEM, one can use a fast moving single wire through the beam and get the beam profile by measuring the position of the wire as a function of the flow of current.

Flying Wire:
Principle: A single wire (of a few micron thickness of carbon) is made to fly through a circulating beam. The scattered particles are detected using scintillators.

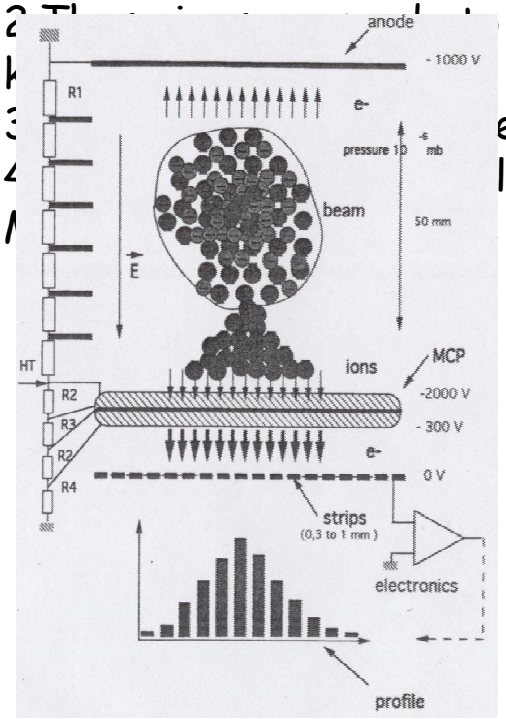


By gating the data this device can be used to measure **bunch by bunch beam profile** in circular accelerators.

Beam Profile Monitors (cont.)

Ion Profile Monitors: (non-distructive device)

Principle: As a charged particle beam circulates in an accelerator
1.It ionizes the residual gas molecules in the beam pipe. (The typical vacuum of an accelerator is ~10^{-7} to 10^{-8} Torr).

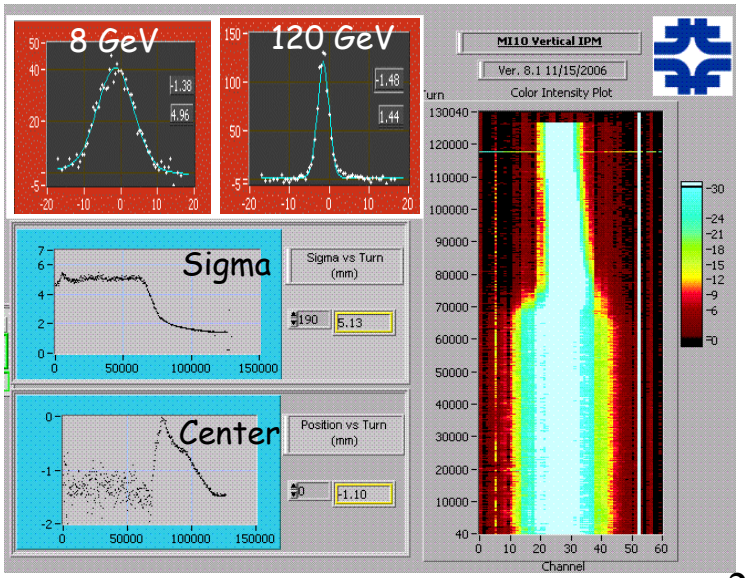


accelerate towards a micro-channel plate which is at a few

es release many electrons for each ions that stricks it.
llected in an anode strip which gives beam profile similar to

Example:

Beam profile data taken during the beam acceleration in the Fermilab Main Injector from 8-120 GeV



Beam Profile Monitors (cont.)

Beam Scrapers:

Principle: The beam loss data from an incremental destruction of the beam in a storage ring using scraper can be used to determine the beam size.

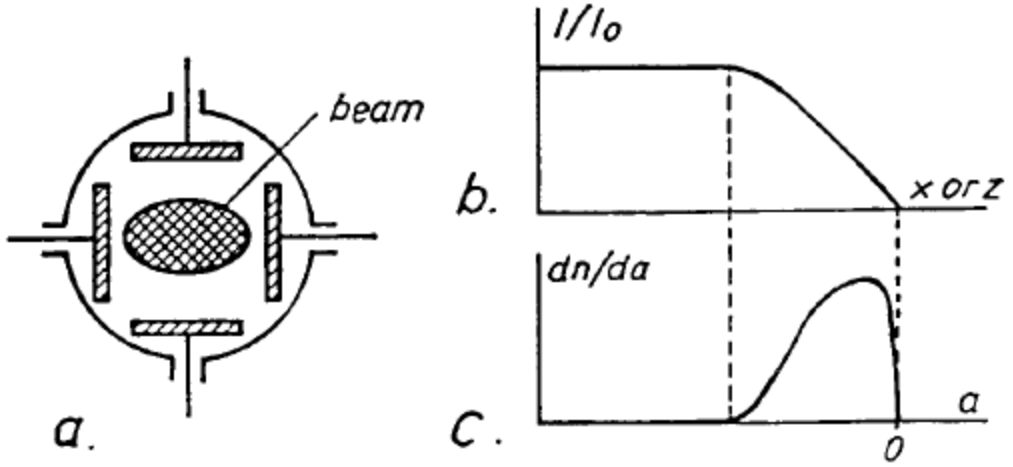


Fig. 26 a) Scraper with four blades for horizontal and vertical measurement. b) Beam intensity vs. blade position. c) Amplitude distribution.

Beam Loss Monitors (BLM)

These devices are used to monitor beam losses during beam transport, storage and acceleration. They are quite critical in tuning the machines and protecting the sensitive devices in the accelerator.

BLMs are indispensible devices in the safety chains.

These come in many varieties. For example,
1.Ionization Chambers
2.Scintillator plus photomultiplier

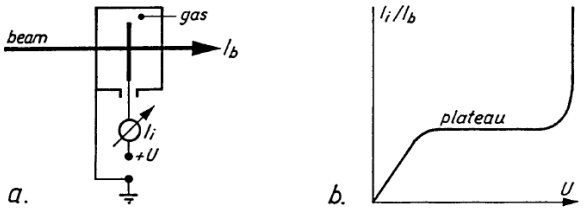
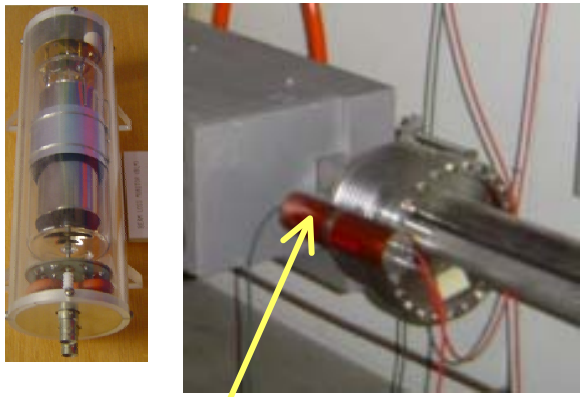
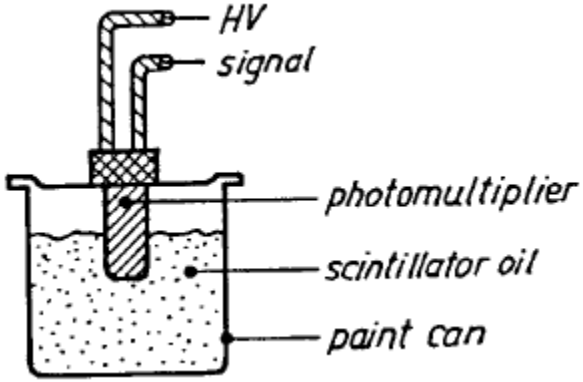


Fig. 20 a) Ionization chamber. b) Collection efficiency vs. voltage.

Ionization Chambers:
Principle: This is a gas filled thin-walled chamber with a collector electrode inside. Typically, one uses argon gas. As a particle passes through it the gas gets ionized and the electrons move towards the anode and current can be measured. This phenomenon can be used to detect beam loss in an accelerator.


Beam loss monitor in
MI-RR pbar extraction LAM



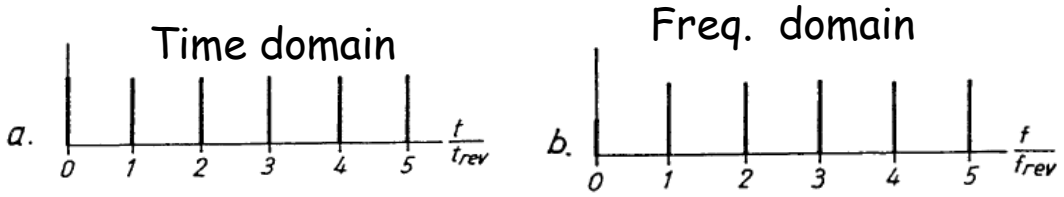
Photomultiplier immersed in a scintillator oil is a very inexpensive but very effective and fast BLM.

Schottky Detectors

Principle: A single particle in a circular machine would produce a delta-function type voltage pulse on a pickup electrode once every revolution period. In the frequency domain, this appears as set of lines equally spaced with a spacing of revolution frequency. If there is more than one particle with an energy spread then in time domain it looks like random distributions of particles in the machine (hence name Schottky signal). On the other hand in the frequency domain, it looks like line equally spaced pulses with spacing of revolution frequency.

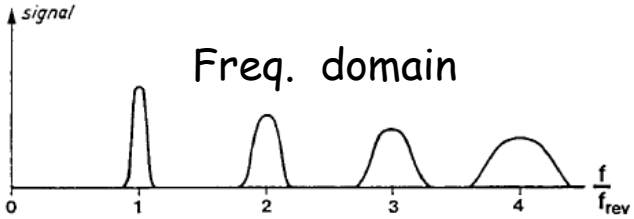
Longitudinal Schottky Signals

Single Particle in a circular machine



Many Particle (with $\Delta p \neq 0$) in a circular machine

Just noise



Following features of the Schottky spectrum is quite relevant to beam diagnostics

1. Area $\propto$ the beam intensity and is same for all bands
2. Band width $\propto$ $h = \dfrac{f}{f_{rev}}$ with $\dfrac{\Delta p}{p} = \dfrac{1}{\eta}\dfrac{\Delta f}{f}$; can be used to measure Δp

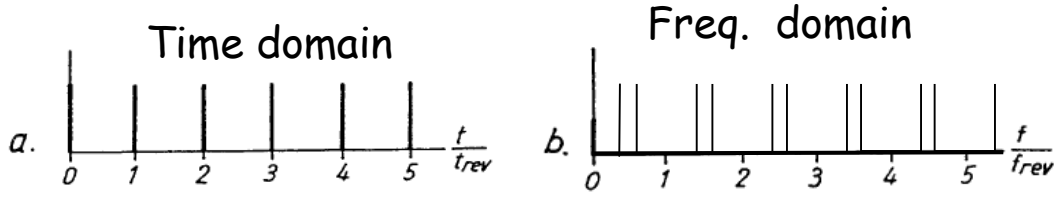
Schottky Detectors (cont.)

Transverse Schottky Signals

Principle: The particles in a circular machine have also got betatron oscillations about the closed orbit with an oscillation frequency=tune of the machine. Consequently, the Fourier analysis of the signals from an **ideal** position sensitive pickup detectors should show two peaks symmetric to the central peak with their spacing such that

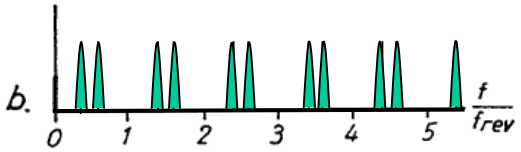
$$f_{Schottky} = (n \pm \nu) f_{rev} \quad \text{where } n = \text{harmonic number of the peak and } \nu = \text{fractional tune}$$

Single Particle in a circular machine with betatron oscillation



Many particles with a tune spread

Just noise



Following features of the transverse Schottky spectrum is quite relevant to beam diagnostics

1. Area under the side band $\propto$ emittance of the beam
2. The distance from the center $\propto$ fractional tune of the beam
3. Width of the side band $\propto$ tune spread

Schottky Detectors (cont.)

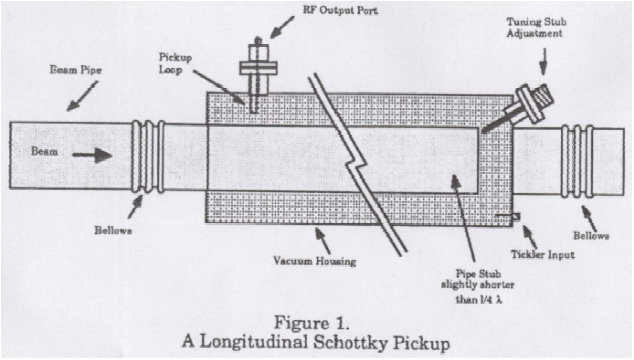


Figure 1.
A Longitudinal Schottky Pickup

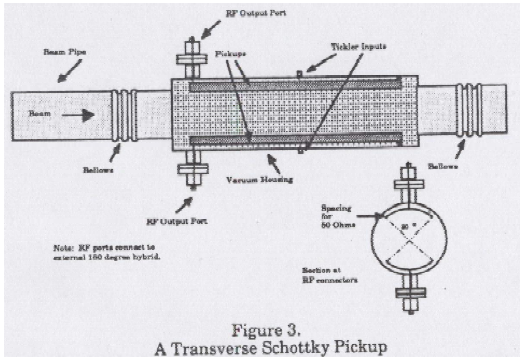
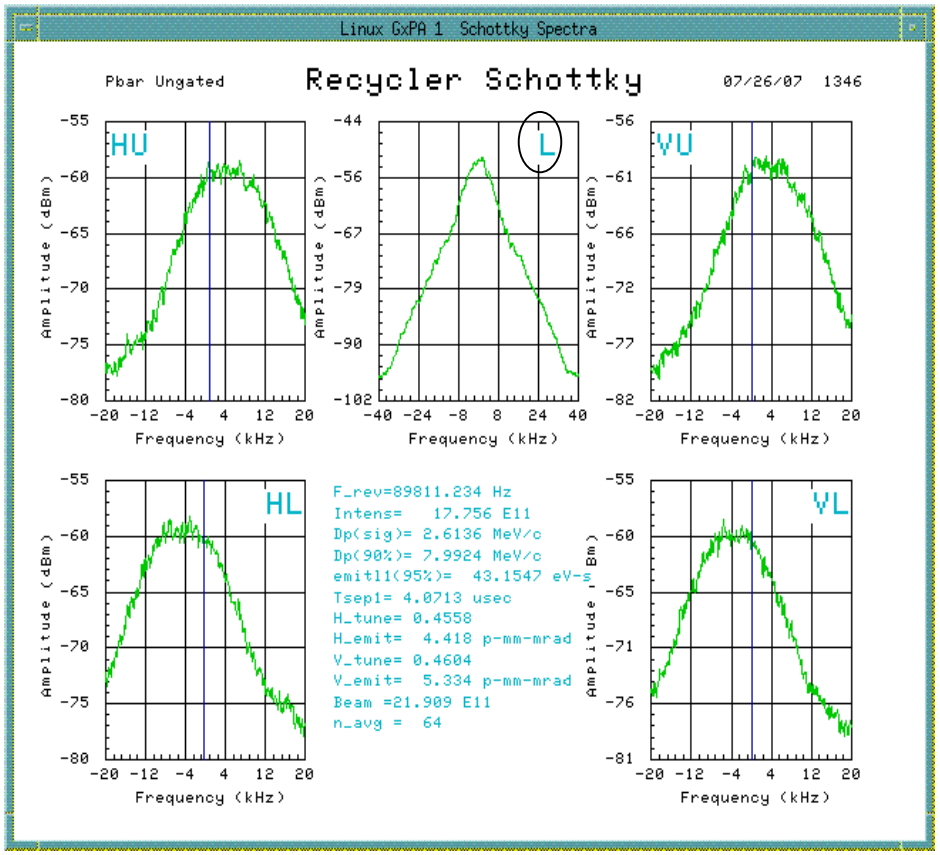


Figure 3.
A Transverse Schottky Pickup



HU
VL
HL
VL
} Transverse Schottky data

Area of these distributions ∝ Transverse emittance

L → Longitudinal Schottky data

Area of these distributions ∝ Beam Intensity

RMS width ∝ Energy spread

Lecture 3 (part 3)

Practical Issues
Closed orbit, Dispersion Function, Chromaticity, Resonances

Chandra Bhat

Dynamics of Particle Beam with Momentum Spread

So far we have investigated dynamics for monochromatic charged particle beam; i.e., with $\Delta p/p=0$. In reality, the beam will have finite momentum spread. Now, we will extend the formalism for a particle beam with $\Delta p/p \neq 0$.

The equation of motion we need to use is the inhomogeneous Hill's equation,

$$u''(s) + K(s)u(s) = \frac{\delta}{\rho} \qquad (1)$$

we will drop the subscript on ρ, anyhow it is a function of "s"

In this case, the particles with relative momentum deviation are "less" or "more" strongly bent than the reference particle. Consequently, they move about a different orbit that deviates from the reference orbit.

The general solution to this differential equation may be written as shown below,

$$u(s) = u_h(s) + u_i(s) \qquad (2)$$

$u_h(s)$: Complete solutions to homogeneous Hill's equation

$u_i(s)$: Particular solution to in-homogeneous Hill's equation

It is obvious that, for a fixed Δp ($\delta = \Delta p/p$) if u_i is a solution, then nu_i will be a solution to $n\Delta p/p$. Therefore, we normalize u_i w.r.t. δ, i.e.,

$$\frac{u_i(s)}{\delta} = D_i(s) \qquad (3)$$

The function D(s) is called **"dispersion"** function.

and write our general solution as,

$$u(s) = u_h(s) + D(s)\delta = C(s)u_0 + S(s)u_0' + D(s)\delta \qquad (4)$$

One assumes that the radial distance from the design orbit is proportional to δ

Beam with Momentum Spread (cont.)

Now by substituting for u(s) in Eq. 1 and equating the homogeneous part to zero, we get,

$$D''(s) + K(s)D(s) = \frac{1}{\rho}$$

The function D(s) is defined as a particular solution for the inhomogeneous equation — 5

This describes the momentum-dependent part of the motion. It is important to note that there is no dispersion function unless there is at least one dipole magnet in the beam line.

The initial conditions for the homogeneous equation were,

$$C_0 = C(s = s_0) = 1,\ S_0 = S(s = s_0) = 0,\ C'_0 = C'(s = s_0) = 0,\ S'_0 = S'(s = s_0) = 1$$ — 6

For dispersion function we chose the initial conditions as ,

$$D_0 = D(s = s_0) = 0 = D'_0 = D'(s = s_0)$$

u(s) and *u'(s)* are related to their initial values by a linear transformation as discussed earlier,

$$\begin{bmatrix} u \\ u' \end{bmatrix}_{s=s} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \begin{bmatrix} u \\ u' \end{bmatrix}_{s=s_0} + \delta \begin{bmatrix} D \\ D' \end{bmatrix}_{s=s_0} \quad \text{or} \quad \begin{bmatrix} u \\ u' \\ \delta \end{bmatrix}_{s=s} = \begin{bmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ u' \\ \delta \end{bmatrix}_{s=s_0} = M \begin{bmatrix} u \\ u' \\ \delta \end{bmatrix}_{s=s_0}$$ — 7

It can be shown that the dispersion function can be calculated from

$$D(s) = S(s)\int_{s_0}^{s} \frac{C(t)}{\rho(t)}dt - S(s)\int_{s_0}^{s} \frac{S(t)}{\rho(t)}dt$$ — 8

Beam with Momentum Spread (cont.)

(Transformation Matrices)

Drift Space: $\frac{1}{\rho_0} = 0, \quad K = 0$

$$M = \begin{bmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

9

Pure Quadrupole: Dispersion = *0* since $1/\rho = 0$ and $\theta = l/k$

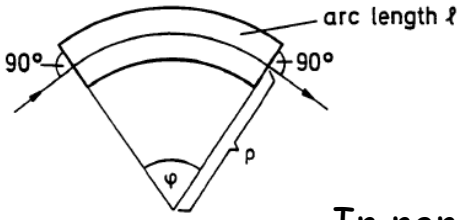
horizontal focusing

$$M_{QF} = \begin{bmatrix} \cos\theta & \frac{\sin\theta}{\sqrt{k}} & 0 \\ -\sqrt{k}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad M_{DF} = \begin{bmatrix} \cosh\theta & \frac{\sinh\theta}{\sqrt{k}} & 0 \\ \sqrt{k}\sinh\theta & \cosh\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

horizontal defocusing

10

Dipole Sector Magnets: $k = 0$ no focusing $\varphi = l/\rho$



$$M_H = \begin{pmatrix} \cos\varphi & \rho\sin\varphi & \rho(1-\cos\varphi) \\ -\frac{1}{\rho}\sin\varphi & \cos\varphi & \sin\varphi \\ 0 & 0 & 1 \end{pmatrix}; \quad M_V = \begin{pmatrix} 1 & \ell & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

11

In non-deflecting plane the magnet behaves like a drift space.

Similar transformation can be derived for FODO cells including dispersion

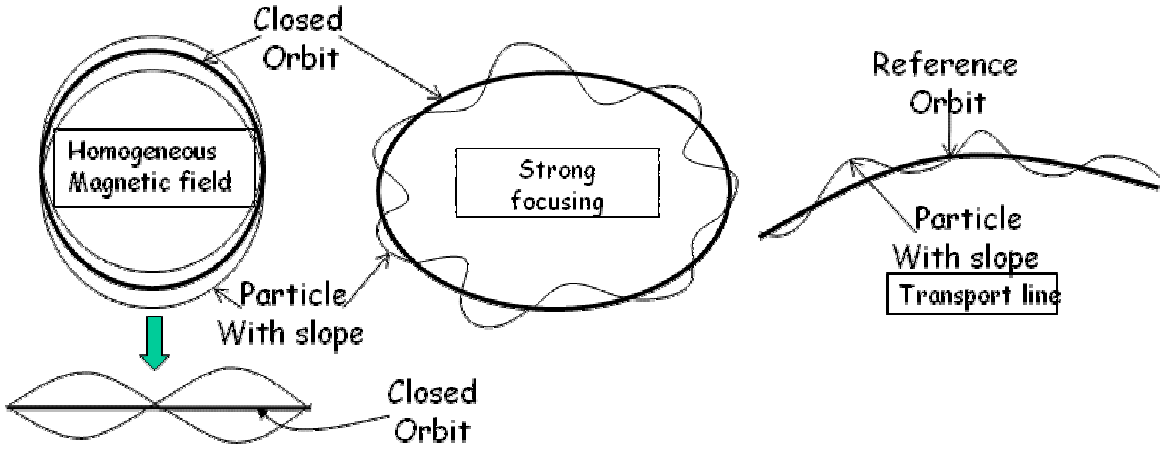
closed orbit

The design orbit of a circular machine is a closed curve that goes through the center of all quadrupoles. This means, a particle with nominal momentum p_0 that starts at some point with

$u_0=0$ and

$u'_0=0$ *radian*

will move on this orbit for arbitrary number of revolutions. Such an orbit is called "**closed orbit**". In case of a beam transport system this is called "**reference path**".



In an accelerator, the transverse motion is divided into

1.Closed orbit which closes on itself

2.Small amplitude betatron motion around the closed orbit.

In any case, the closed orbit need not have a well-defined shape because accelerators have a number of horizontal and vertical dipole correctors.

However, a particle with $p=p_0$ but with "**non-zero displacement**" and/or slope with respect to the closed/reference orbit will conduct betatron oscillations.

Closed-Dispersion Trajectory

Now consider a particle with $p \neq p_0$. For this particle the closed orbit is displaced from the ideal closed orbit. Also, we recall that the radial distance of this particle is proportional to δ.

$$u_D(s) = D(s)\delta \qquad (12)$$

$u_D(s)$ is called "off-momentum closed orbit". In weak focusing machine the dispersion is constant, while, for a strong focusing case the dispersion is "s" dependent.

A particle with $p \neq p_0$ satisfies the inhomogeneous Hill's equation and the total deviation of the particle from the reference orbit is written as,

$$u(s) = u_\beta(s) + u_D(s) \qquad (13)$$

$u_\beta(s)$: describes betatron oscillations around

$u_D(s)$: deviation of the closed orbit for off momentum particle

Since $K(s)$ is a periodic function of s, we impose $D(s)$ and $D'(s)$ to be periodic as follows,

$$D(s+C) = D(s)$$
$$D'(s+C) = D'(s) \qquad (14)$$

In literature, a periodic dispersion is often denoted by $\eta(s)$

The closed orbit condition for the dispersion function is strictly required only for one complete revolution, where C is circumference of the machine. Though the local periodic closed orbit condition is not necessary, it helps for lattice design.

Remark: For each particle energy only one equilibrium orbit exists in a given closed lattice.

Closed Dispersion Trajectory (cont.)

Without getting into the detailed derivation we simply state that the closed periodic dispersion function can be written as,

$$\eta(s) = D(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu} \int_{s}^{s+C} \frac{\sqrt{\beta(t)}}{\rho(t)} \cos[\nu(\phi(s) - \phi(t) + \pi)]dt$$

ν is the tune of the circular machine.

15

The dispersion function written in the above form has two important implications:

1. The particle trajectory receives an additional kick only inside the bending magnet, i.e., $\rho(t) \neq 0$
2. To get stable orbit the tune $\nu \neq$ integer.

The second implication clearly states that "a finite dispersion exists only if the number of betatron oscillations per revolution is different from an integer."

Physical interpretation of integer ν:

A particle with Δp will receive a different kick angle as compared to a particle on the closed orbit as it enters a dipole in a ring. If ν is an integer then this particle arrives at the same point with angular deviation from the previous kick angle and adds up coherently from turn to turn. Soon the deviation of the path of the particle becomes too large to be confined within the beam pipe. This brings about the fundamental phenomenon of **resonances** in accelerator physics.

Remark: The dispersion function as presented above is very useful for understanding the oscillatory behavior of closed orbits in presence of dipole errors or momentum errors. However, numerical calculations of dispersion function will be easier with matrix formalism.

Closed Dispersion Trajectory (cont.)

The transformation matrix for the dispersion function can be obtained by substituting Eq. 13 in 7 and with some mathematical arrangements we get,

$$\begin{bmatrix} \eta\delta \\ \eta'\delta \\ \delta \end{bmatrix}_{s=s} = \begin{bmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta\delta \\ \eta'\delta \\ \delta \end{bmatrix}_{s=s_0} \quad \text{or} \quad \begin{bmatrix} \eta \\ \eta' \\ 1 \end{bmatrix}_{s=s} = \begin{bmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta \\ \eta' \\ 1 \end{bmatrix}_{s=s_0} \qquad 16$$

Once the η and η' have been determined at one point s_0, the values at any point can be calculated.

Using the periodic closed orbit conditions applied to the circular accelerators we can show that,

$$\eta = \frac{(1-S')+SD'}{2-S'-C} = \frac{(1-S')+SD'}{4\sin^2 \pi\nu}$$

$$\eta' = \frac{C'D+(1-C)D'}{2-S'-C} = \frac{C'D+(1-C)D'}{4\sin^2 \pi\nu} \qquad 17$$

Again integer tune values are to be avoided.

Beam Size:

We have, $u(s) = u_\beta(s) + D(s)\delta$ for the trajectory of a particle. Averaging the square of this expression and finding RMS value, we find the rms size of the beam due to betatron motion and momentum spread looks like

$$\sigma^2 = \frac{\varepsilon\beta}{\pi} + D^2\langle\delta^2\rangle \qquad 18$$

A reminder: We have to remember that $u(s)$ stands for both x or y plane. Hence , we have to deal with betatron function, dispersion and beam sizes in both horizontal and vertical planes separately.

Most General Formalism

The most general form of transformation matrix including both horizontal and vertical degrees of freedoms becomes,

$$\begin{bmatrix} x \\ x' \\ \delta \\ y \\ y' \end{bmatrix}_s = \begin{bmatrix} C_x & S_x & D_x & 0 & 0 \\ C_x' & S_x' & D_x' & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & C_y & S_y \\ 0 & 0 & 0 & C_y' & S_y' \end{bmatrix} \begin{bmatrix} x \\ x' \\ \delta \\ y \\ y' \end{bmatrix}_{s_0} \qquad (19)$$

Here we assume that there is no dispersion in vertical plane. This is typically true in most of the horizontal circular machines.

Momentum Compaction Factor in Transverse Dynamics

Consider a particle with a fixed momentum p_0 which moves along a closed orbit in magnetic field. Then the following relations can be written,

$$\oint \frac{ds}{\rho} = 2\pi, \quad \frac{1}{\rho} = \frac{q}{p_0} B_{Perpendicular} \quad \text{and} \quad p_0 = \frac{q}{2\pi} \int B_{Perpendicular} ds \qquad (20)$$

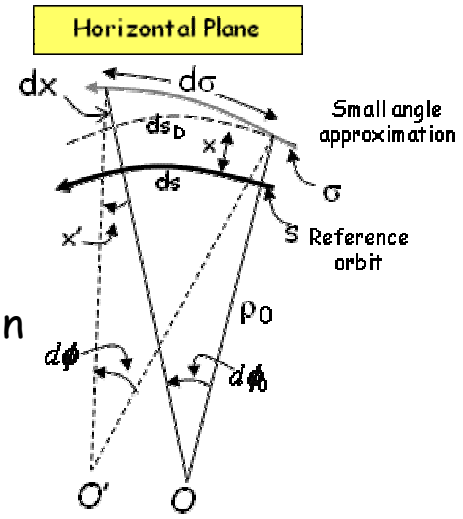
Now introduce another particle with a momentum offset of Δp. Its closed orbit must be different from that of with p_0. By varying the p, the length of the closed orbit can be varied, i.e.,

$$L = \oint ds$$

can be changed. We know that, the differential trajectory length is given by,

$$d\sigma = ds\left[1 + \frac{x_D}{\rho}\right] \quad \text{or} \quad L + \Delta L = \oint d\sigma = \oint \left[1 + \frac{x_D}{\rho}\right] ds$$

$$\text{or} \quad \Delta L = \oint \frac{x_D}{\rho} ds$$



In this case ΔL corresponds to the closed orbit dispersion trajectory, $x = D(s)\delta = D(s)\frac{\Delta p}{p}$ and substituting in above equation we get,

$$\frac{\Delta L}{L} = \frac{1}{L}\oint \frac{D(s)}{\rho} ds \bullet \frac{\Delta p}{p} \Rightarrow \frac{\left[\frac{\Delta L}{L}\right]}{\left[\frac{\Delta p}{p}\right]} = \boxed{\alpha_c = \frac{1}{L}\oint \frac{D(s)}{\rho} ds}$$

Momentum compaction factor

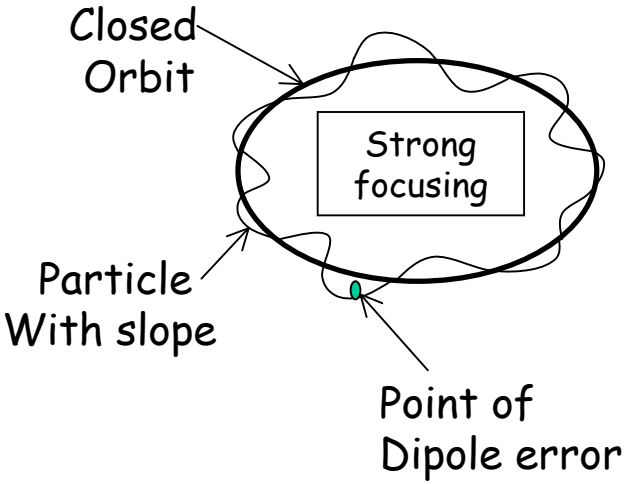
Momentum compaction factor & Dispersion function

Linear Deviations from the Ideal Lattice

So far we have established the basic principles underlying the single particle accelerator design. In reality, we have to deal with electric and magnetic systems having non-uniformity (called systems errors).

Dipole errors or Steering errors:

For a circular accelerator there is a closed orbit (ideal orbit) by design. For a particle with non-zero emittance (i.e., $x \neq 0$, or $x' \neq 0$ or both) we expect betatron oscillations about the closed orbit. Suppose that a particular dipole magnet in one of its arcs has a field slightly different from its intended value and gives a single steering error,



$$\alpha = \frac{\Delta B}{p} l = \frac{\Delta B}{B\rho} l \qquad (21)$$

ΔB is unintentional uniform field over length "l" which is different from everywhere else (treated as an error).

Now let (x_0, x_0') be the coordinates of a particle (which would follow reference orbit or has a betatron oscillations) just outside of the special dipole. Let M represent the single turn transfer matrix for the ring. Then, for stability of the particle we demand that after the particle goes through the error region the coordinates do not change. This means,

$$\begin{bmatrix} x_0 \\ x_0' \end{bmatrix} = M \begin{bmatrix} x_0 \\ x_0' \end{bmatrix} + \begin{bmatrix} 0 \\ \theta \end{bmatrix} \Rightarrow \begin{matrix} x_0(\cos\phi + \alpha_0 \sin\phi - 1) + x_0' \beta_0 \sin\phi = 0 \\ x_0(-\gamma \sin\phi) + x_0'(\cos\phi + \alpha_0 \sin\phi - 1) = \theta \end{matrix} \quad \text{with } \phi = 2\pi\nu \qquad (22)$$

Dipole Errors(cont.)

Solving for x_0 and x_0', we get,

$$x_0 = \frac{\theta\beta_0 \cos\frac{\phi}{2}}{2\sin\frac{\phi}{2}} = \frac{\theta\beta_0 \cos \pi\nu}{2\sin \pi\nu}$$

$$x_0' = \frac{\theta[\sin \pi\nu - \alpha_0 \cos \pi\nu]}{2\sin \pi\nu}$$

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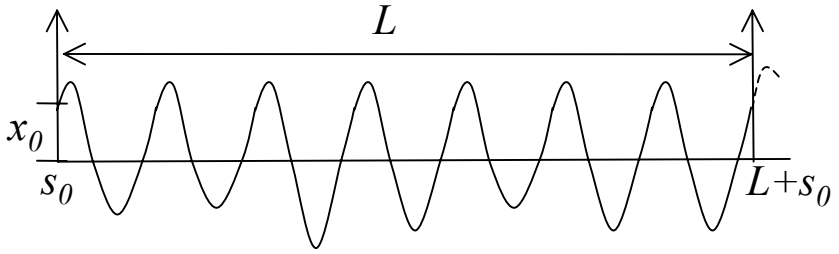

These equations also show that dipole error in an accelerator ring prohibit integer tunes for beam to be stable.

If the dipole error is distributed around the ring (which is generally the case), then the closed orbit can be obtained by integrating over all dipole kicks.

$$x_{closed}(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\int_s^{s+C}\sqrt{\beta(t)}\frac{\Delta B}{B\rho}\cos[2|(\phi(t)-\phi(s)|+\pi\nu)]dt$$

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Conclusions: the dipole errors and off momentum particles make integer tunes forbidden in a circular accelerators.

Quad Errors

Quadrupole Errors:

Let $K_0(s)$ be the design quadrupole strength and ΔK be the quad error at a point in the ring. The with $\Delta p=0$ the equation of motion becomes,

$$u''+[K_0(s)+\Delta K(s)]u=0 \qquad (25)$$

If this error is local and at one point in the ring then it can be shown that the change in phase advance at that cell is

$$\cos\phi-\cos\phi_0=-\frac{1}{2}\beta\,\Delta K\,ds_1\sin\phi_0 \qquad (26)$$

error is occurring at $s=s1$

If the perturbation is distributed around the ring, then the change in tune is given by,

$$\Delta\nu=\frac{\Delta\phi}{2\pi}=\frac{1}{4\pi}\oint\beta(s)\Delta K(s)ds \qquad (27)$$

Further, a gradient error changes beta function as follows,

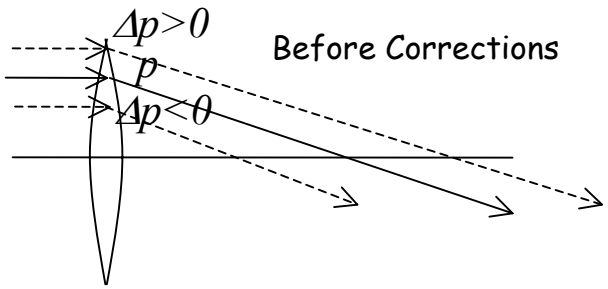
$$\Delta\beta(s)=\frac{\beta(s)}{2\sin 2\pi\nu}\oint\beta(t)\Delta K\cos[2|(\phi(t)-\phi(s)|+2\pi\nu)]dt \qquad (28)$$

Thus, as $\nu\rightarrow n/2$ with $n=$ integer $\Delta\beta\rightarrow\infty$ or the beam becomes unstable.

Conclusions: The quadrupole errors make half integer tunes forbidden in circular accelerators.

Chromaticity

The particles with different energies in a beam will be focused differently dependent on their momentum. The lowest order chromatic perturbation is caused by the variation of the focal length of the quadrupoles with particle energy. This leads to a shift in tune of the machine.



We know that for large ρ, the quadrupole strength,

$$K(s) = k(s) + \frac{1}{\rho^2} \approx k(s) = \frac{qg(s)}{p} \qquad (29)$$

$$\therefore \Delta K = \frac{dK}{dp}\Delta p = -\frac{qg}{p^2}\Delta p \approx -\frac{qg}{p}\frac{\Delta p}{p_0} = -K(s)\frac{\Delta p}{p_0}$$

Formally, this is same as a gradient error, discussed earlier in connection with quadrupole errors. Hence, to calculate the shift in tune

$$\Delta\nu = \frac{1}{4\pi}\oint \beta(s)\Delta K(s)ds \approx -\frac{1}{4\pi}\oint \beta(s)K(s)ds\frac{\Delta p}{p_0} = \xi\frac{\Delta p}{p_0} \qquad (30)$$

with

$$\xi = -\frac{1}{4\pi}\oint \beta(s)K(s)ds \qquad (31)$$

The quantity ξ is called "chromaticity" of the machine. For a "linear" lattice this quantity is always negative and ξ is called "natural chromaticity". The chromaticity is integral part of a circular machine.

Examples: Natural chromaticity of Main Injector at Fermilab ≈-33.7, that for Recycler it is -2 (by design)

Chromaticity (cont.)

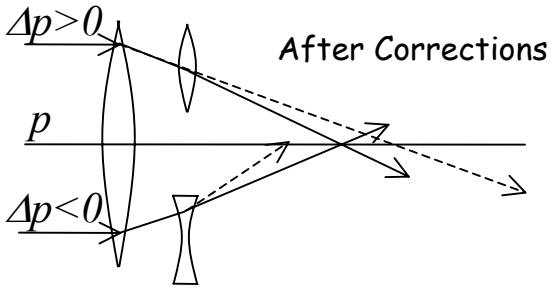
Control of these chromatic perturbations in circular machines is important for the following two reasons

1.Avoid loss of particles due to tune shifts into resonances

2.To prevent beam loss due to "head-tail instability"

The chromatic aberrations are corrected using sextupoles. By means of this system higher energy particles are focused more and lower energy particles are defocused. Thus, all particles are focused at one point.

The consequence of this is the betatron frequency becomes dependent upon the momentum of the particle.

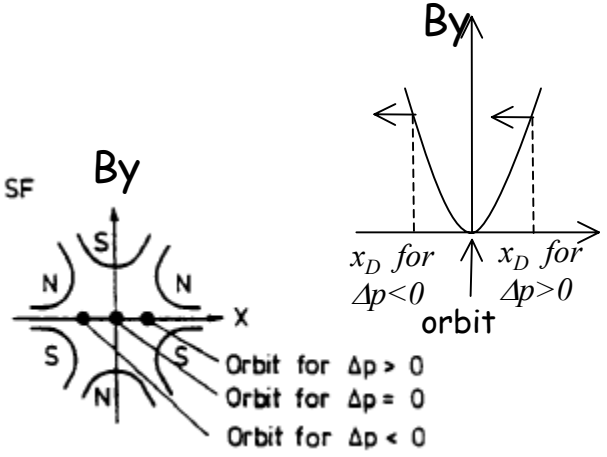


Consider a particle with $\Delta p \neq 0$ moving without betatron oscillations on the closed dispersion trajectory.

$$x_D(s) = D(s)\frac{\Delta p}{p_0} \qquad (32)$$

Now let us introduce a sextupole magnet with $D(s) \neq 0$. The sextupole filed at $x = x_D$ in the horizontal plane is,

$$B_y = \frac{1}{2} g' x^2 \qquad (33)$$



Notice that the particles with $\Delta p = 0$ on the reference orbit are not influenced. With the polarity shown in the figure the sextupole deflects electrons with $\Delta p > 0$ towards the central orbit and those with $\Delta p < 0$ are deflected away from it. Therefore the closed dispersion orbit calculated for a linear machine will change with sextupole magnets in it.

Chromaticity (cont.)

Is this a problem? No! why?

Now consider a particle that executes betatron oscillations around the closed dispersion orbit. Let $\Delta p>0$. Let δx and δy be the small deviation from the closed dispersion orbit. Then,

$$x = x_D(s) + \delta x$$
$$y = y_D(s) + \delta y = 0 + \delta y$$

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The corresponding magnetic field components can be written as,

$$B_y = \frac{g'}{2}(x^2 - y^2) = \frac{g'}{2}[(x_D + \delta x)^2 - \delta y^2] \approx \frac{g'}{2} x_D{}^2 + g' x_D \delta x = \frac{g'}{2} x_D{}^2 + g' D(s) \frac{\Delta p}{p_0} \delta x$$

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Causes deflection of closed **dispersion** orbit

$$B_x = g' xy \approx g' x_D \delta y = g' D(s) \frac{\Delta p}{p_0} \delta y$$

Thus, the sextupole acts like a position dependent quadrupole with its strength increasing with Δp and influences both horizontal and vertical motion, i.e., it **introduces coupling between horizontal and vertical motions**.

The equivalent quad strength arising from the sextupole magnet is

$$q \frac{g' xD}{p_0} = m.D(s) \frac{\Delta p}{p_0}; \quad m = \frac{qg'}{p_0}$$

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By properly choosing its strengths one can compensate for chromaticity.

$$\xi_{comp} = -\frac{1}{4\pi} \oint [K(s) - mD(s)] \beta(s) ds$$

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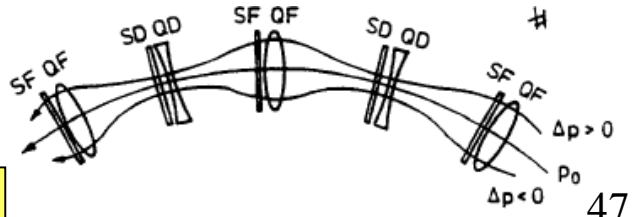


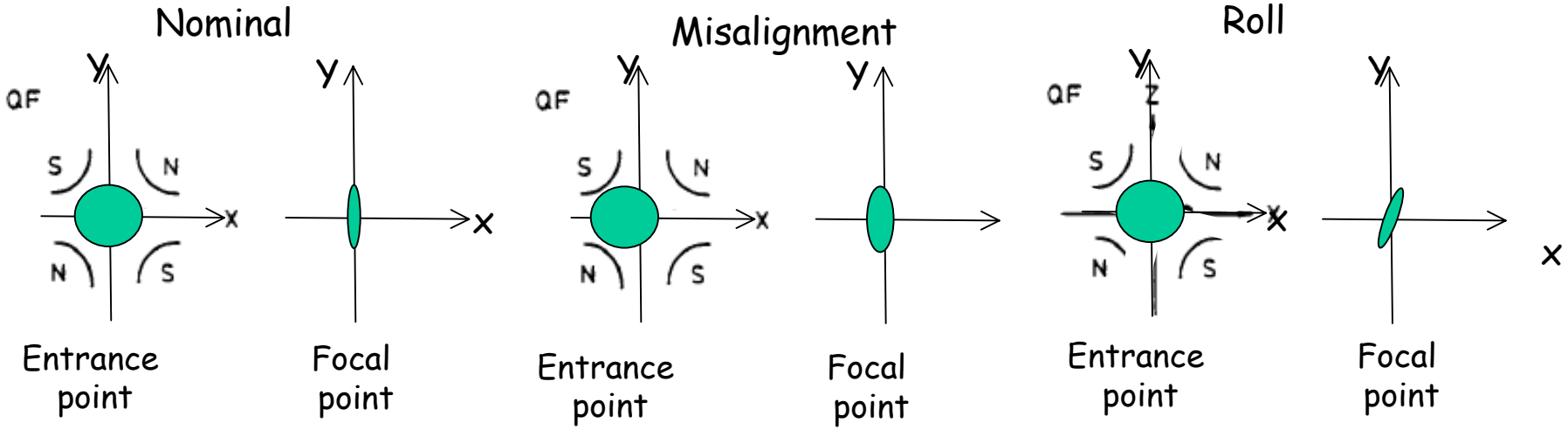
Chromaticity (cont.)

In principle, single sextupole magnet is sufficient to compensate the chromatic effects. In practice a single sextupole magnet introduces aberrations. In order to avoid this, two families of sextupole magnets - one in the H-plane and another in the V plane - are used.

Without any proof we make the following statements:

1."The adverse effect that would be introduced by a single strong sextupole magnet in a circular accelerators can be minimized by using a large number of sextupoles with moderate strength distributed around the ring."

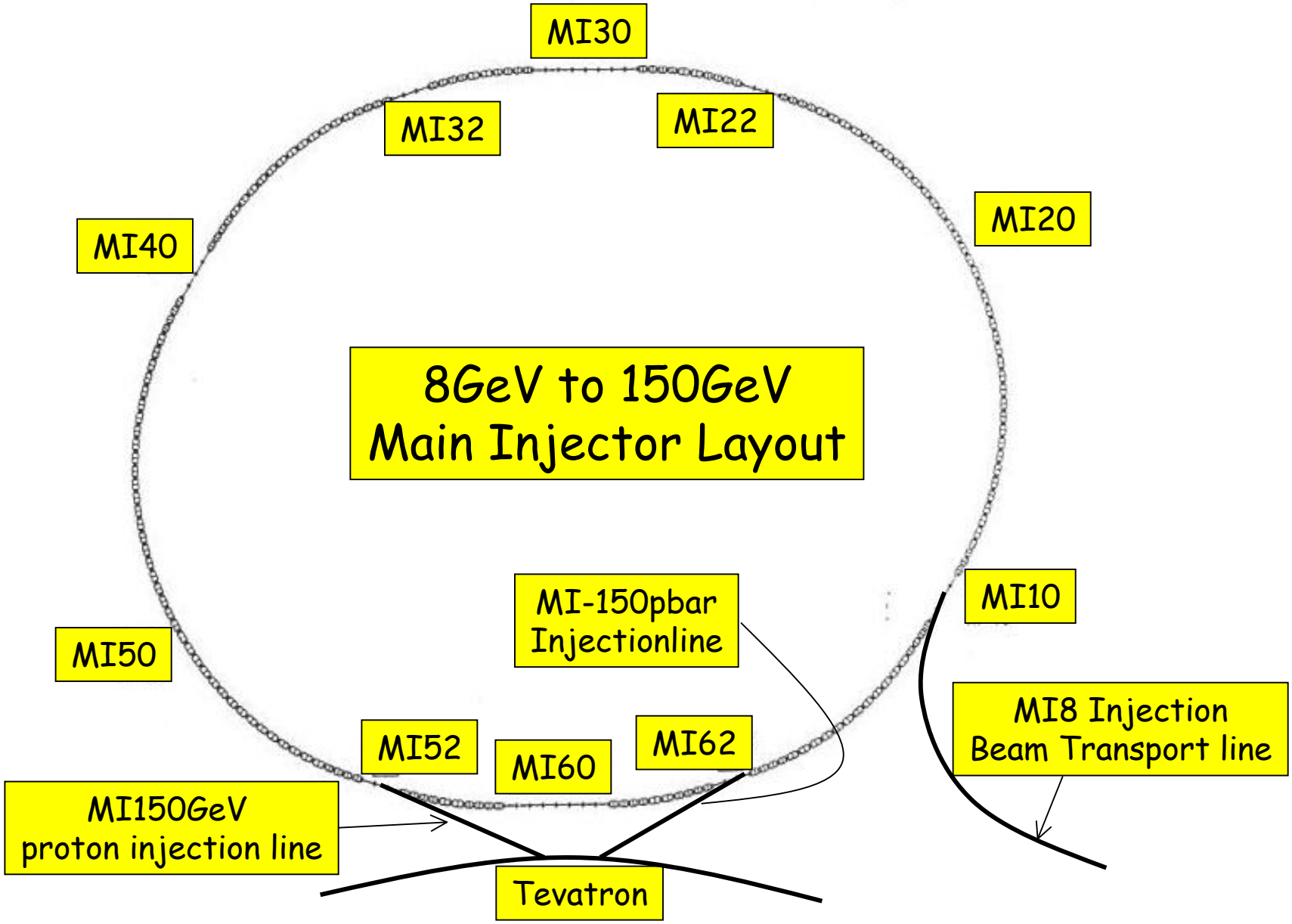
2.The sextupole errors in the ring **introduce resonances at third-integer tunes**.



Positively charged beam into the plane of paper

3. Misalignment and/or role-angle of quadrupoles also **introduces coupling between horizontal and vertical motions.**

Example of Design Lattice



Example(cont.)

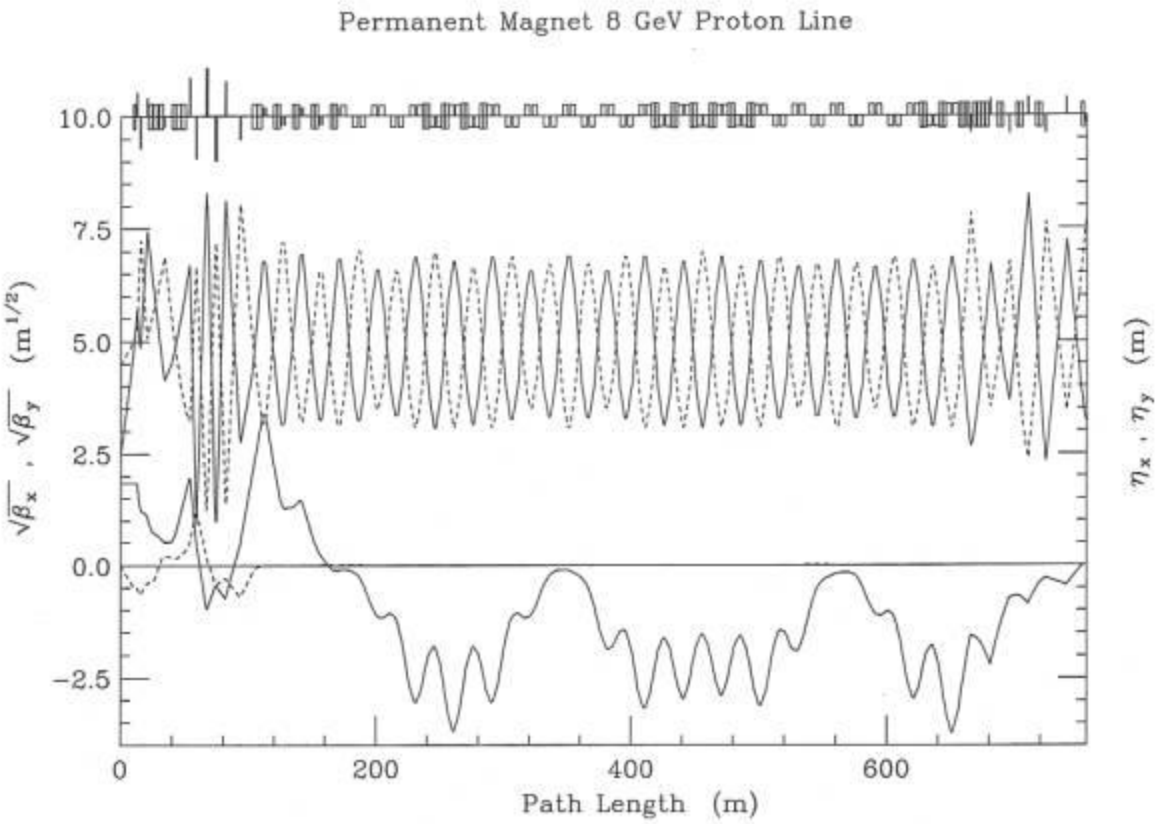


Figure 2.4-2(a). Lattice functions for the 8 GeV line.

Example(cont.)

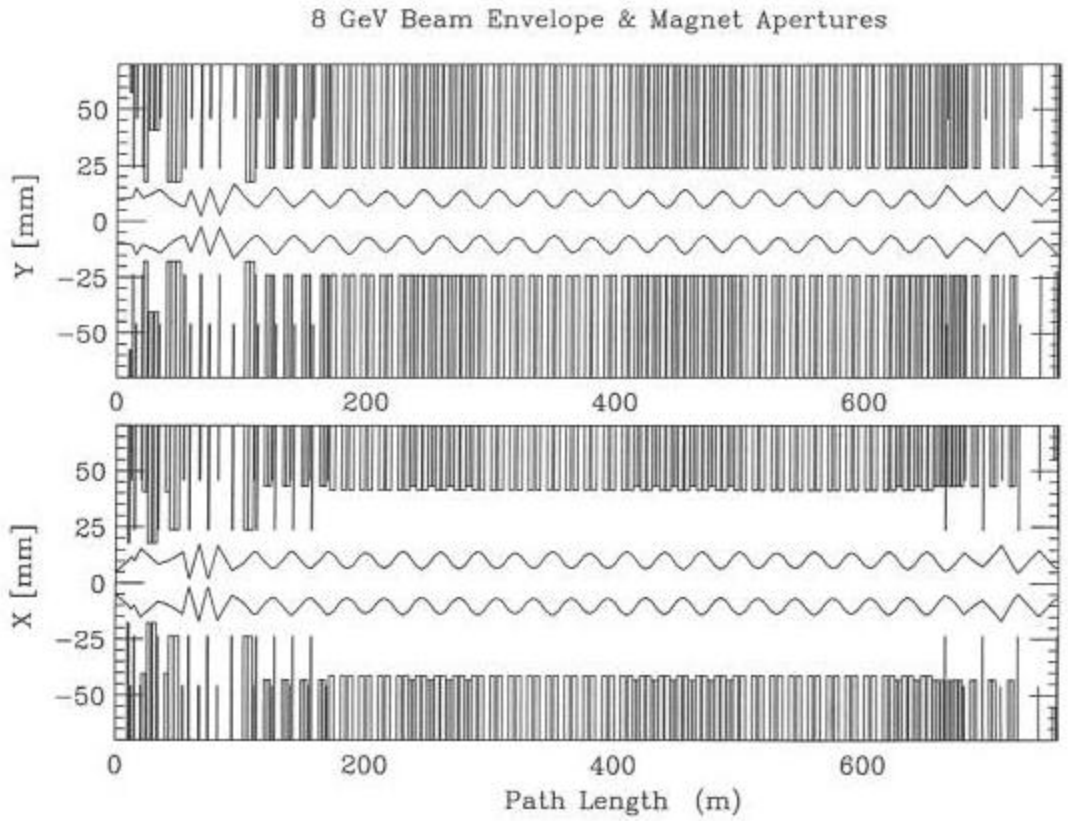


Figure 2.4-2(b). 40π mm-mr beam envelopes and magnet apertures.

Example(cont.)

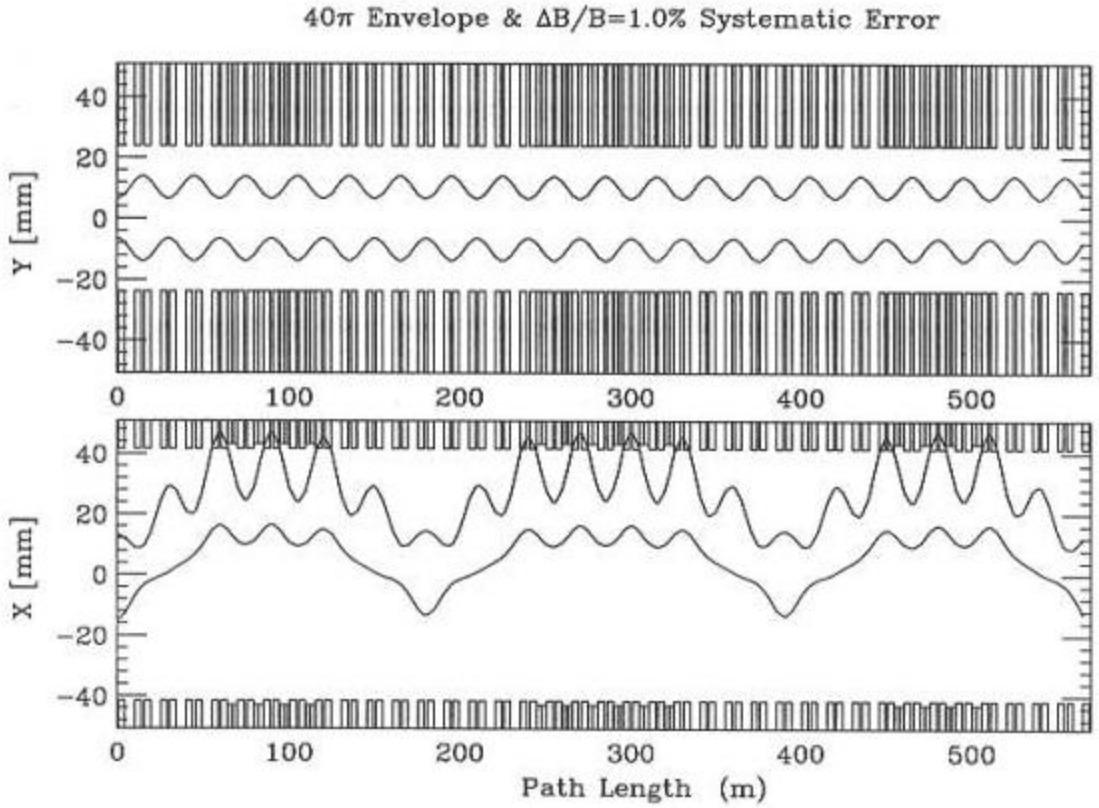


Figure 2.4-4(a). Trajectory and envelope with systematic field error of 1%. momentum spread dp/p = 0.2%.

Example(cont.)

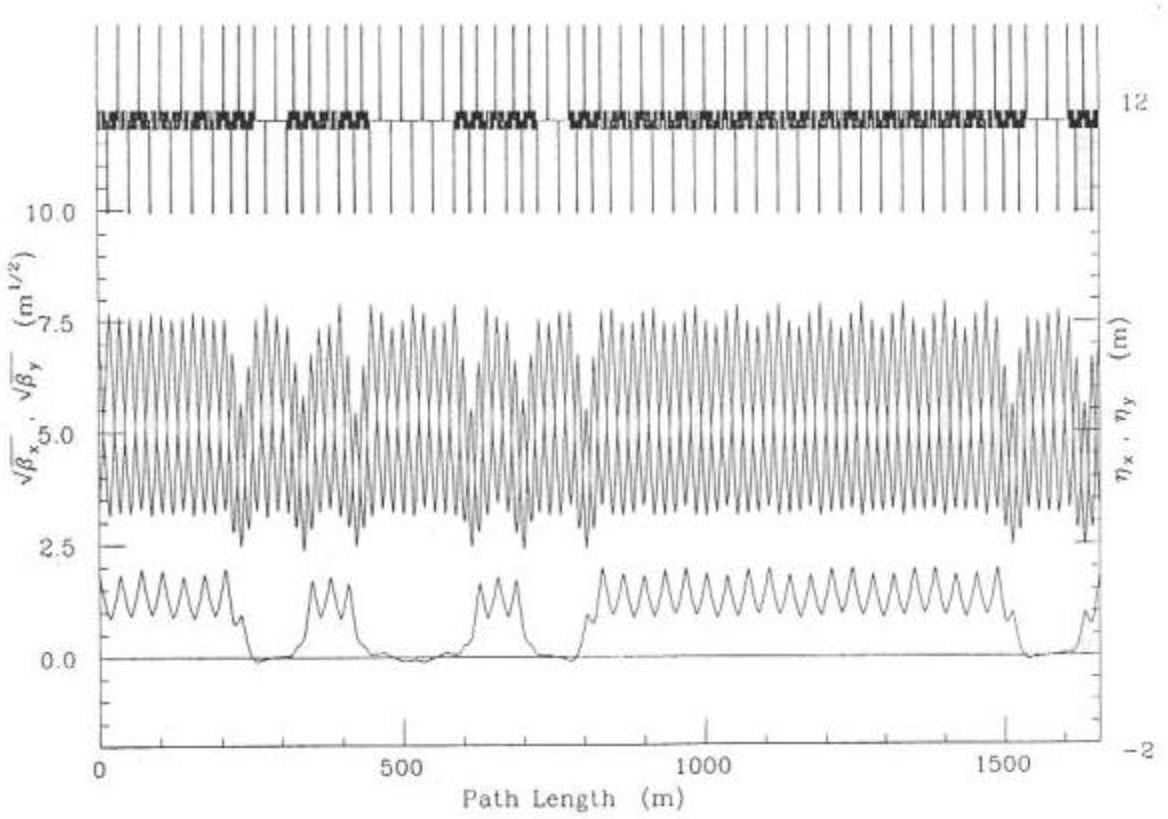


Figure 2.1-3. Main Injector Lattice Functions for One-Half the Ring.

Tune Space and Resonance Diagram

So far we have learnt a few aspects about the resonances in circular machines
1.Dipole errors → integer resonances (i.e. $\nu \neq n$, integer)
2.Quadrupoles → half integer resonances (i.e. $\nu \neq n/2$, integer)
3.Sextupoles → third integer resonances (i.e. $\nu \neq n/3$, integer)

Also, sextupole or misalignment of magnets like skew quadrupole fields give rise to horizontal and vertical coupling.

This leads to a general condition

$$m\nu_x + n\nu_y = l \qquad |m|+|n| = \textbf{order of the resonance.}$$

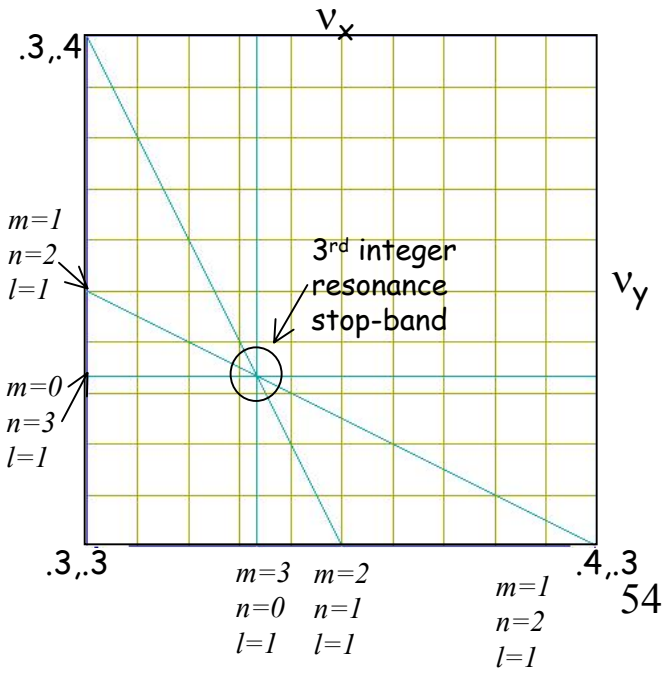
where m, n and l are integers. The operating points (ν_x, ν_y) have to be chosen in a reasonable distance from resonance lines.

Plotting all straight lines for different values of m, n and l in a (ν_x, ν_y) space produces "resonance" diagram.

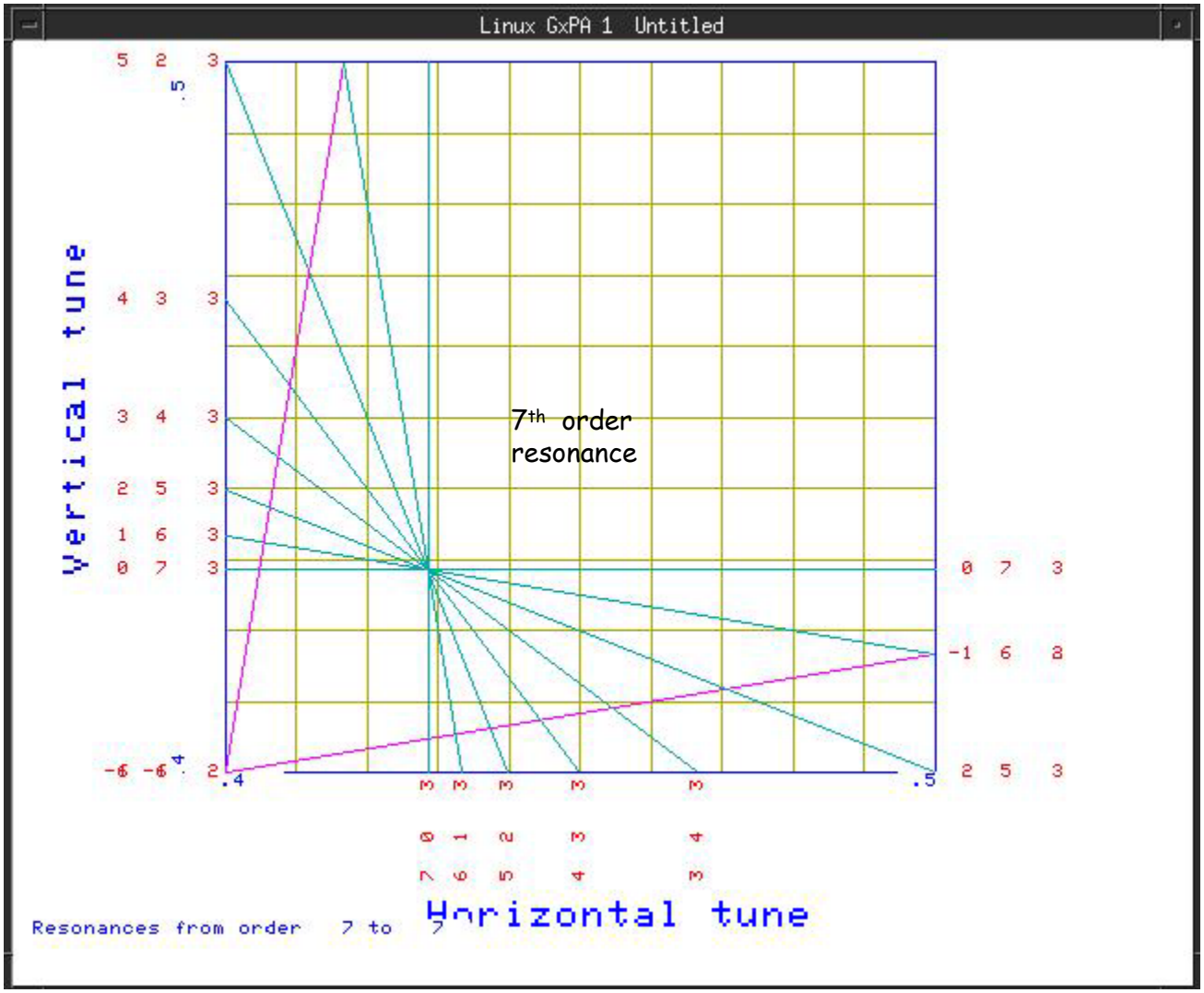
Particles do not survive on integer and half integer resonances at all. But, on other resonances particle may survive.

Stop-band: the region of instability in the resonance diagram

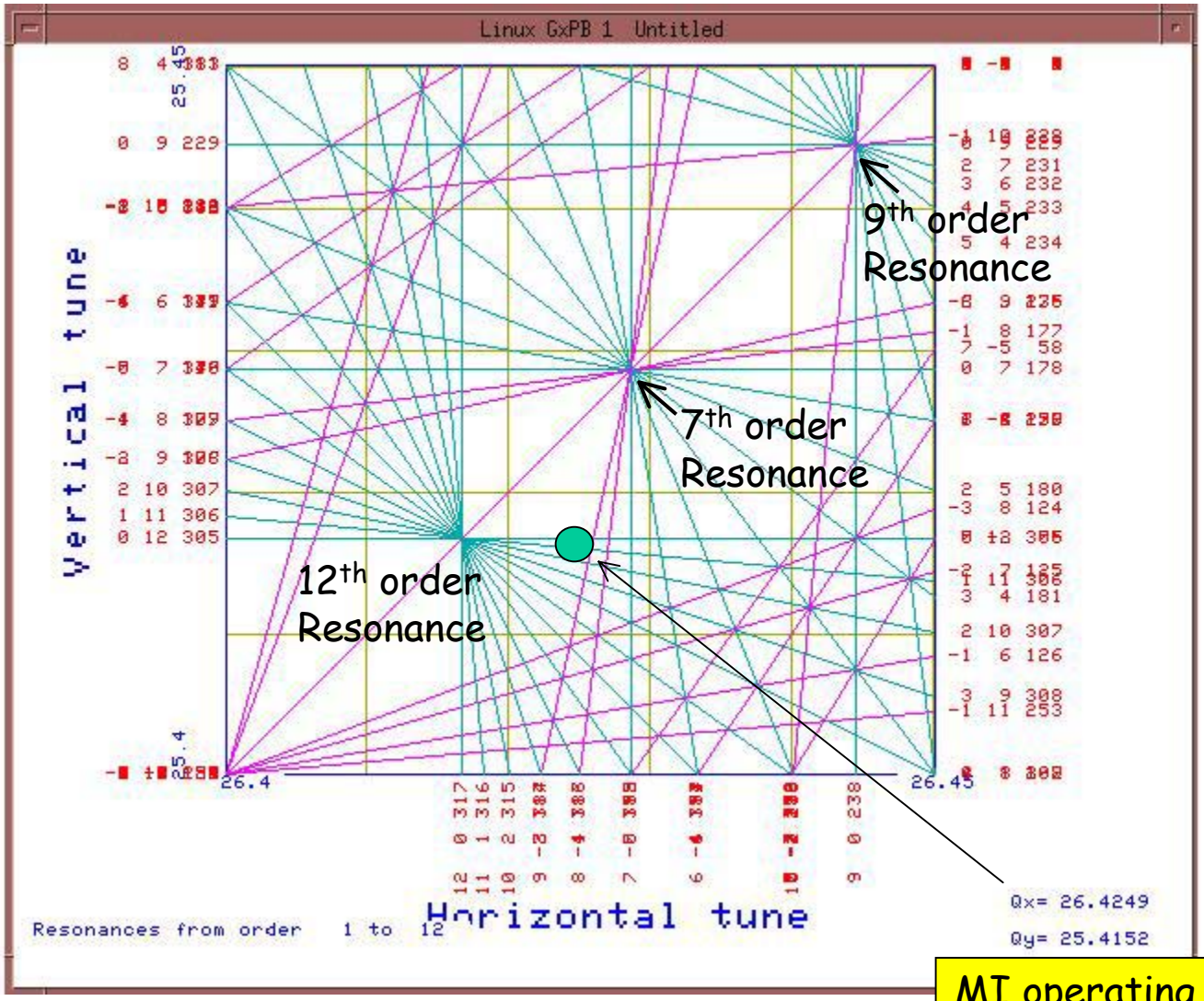
Stop-band width: Width of the unstable tune



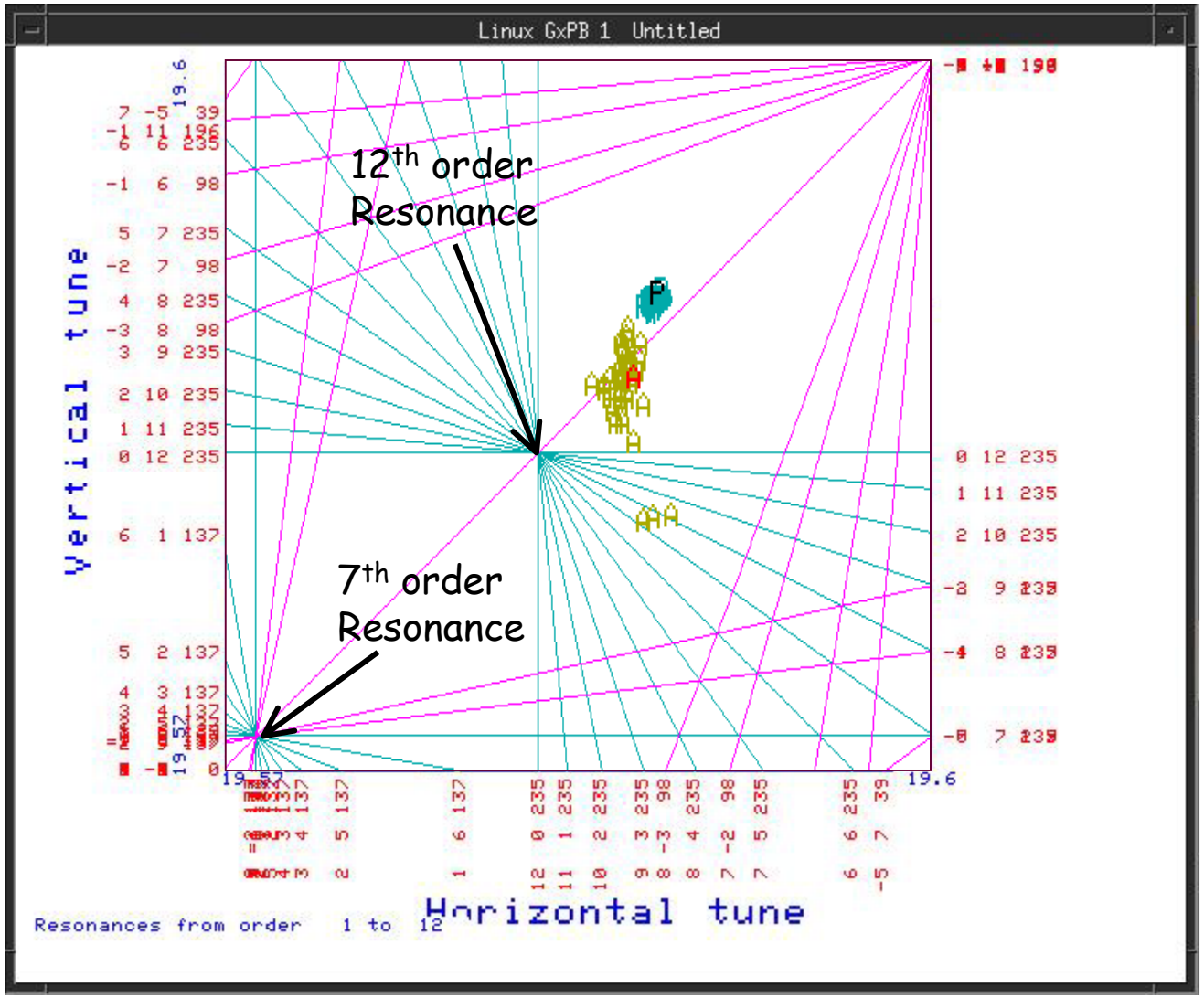
Resonance Diragram (cont)



Main Injector Tune Diagram



Tevatron Tune Diagram



Computer Programs for Lattice Design

Nowadays to design accelerators or beam-lines we do not have to go through these laborious mathematical matrix transformations. To design accelerators and beam lines a number of computer programs are available in the market for free.

1. MAD – methodical accelerator design, ← tracking program
2. SYNCH ??
3. TRANSPORT
4. Turtle
5. TEAPOT ← tracking program